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Unified impulsive fuzzy-model-based controllers for chaotic systems with parameter uncertainties via LMI

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ABSTRACT

In this paper, a novel technique based on impulsive fuzzy T–S model is proposed for controlling chaotic systems with parameter uncertainties. According to this new model, a unified methodology for establishing robust stability, asymptotic stability and exponential stability of impulsive controllers is developed. Various robust stability conditions are presented in the form of linear matrix inequalities (LMI). A simple iterative algorithm is also provided for calculating design parameters based on LMI techniques. Finally, a typical design procedure is developed by using well-known chaotic systems for illustration, accompanied by several numerical simulations to demonstrate the validity of the proposed methodology.

1. Introduction

Due to recent advances made in the applications of chaos to chemical reactions, power converters, biological systems, information processing, secure communications, etc., controlling complex chaotic dynamics for engineering applications has newly emerged as an attractive field accompanied by rapid development in its theories and methodologies [1–17]. For example, linear state feedback [2] and Lyapunov-type methods [3] have been used for linear and nonlinear control-design purposes, respectively. In addition, in [4], a feedback linearization technique has been also applied for controlling chaotic systems. Combining intelligent modeling [5] with chaos control based on fuzzy logic continues to gain more interest in recent years. For example, Tanaka et al. [6] derived chaotic fuzzy models and designed fuzzy-model-based controllers with the aim of stabilizing or synchronizing these chaotic systems. Similarly, impulsive control methods have been also used widely for the same purposes using only small control impulses. While these impulsive control techniques offer a direct method for modulating digital information onto a chaotic carrier signal for spread spectrum applications [7–9], they lack a unified design for controlling different chaotic systems and fail to emulate human operators. It is interesting to note that impulsive techniques have been combined with fuzzy T–S models to design appropriate controllers for nonlinear systems [10–12]. Although qualitative theories available in both domains offer vast resources for analyzing these impulsive fuzzy T–S models, the issue of parameter uncertainties in these com-

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plex nonlinear systems remains unresolved. In practical physical systems, the parameters of chaotic systems may not be known exactly. Their presence may seriously impede system performance, decrease the speed of response and possibly cause chaotic perturbations in the regular behavior of the system if these controllers are not designed properly. Therefore their effects cannot be neglected when designing fuzzy T–S models based on chaotic systems with parameter uncertainties. There are several attempts in the literature to study their effects. For instance, in [13], an input–output control scheme was used for chaos suppression in a class of uncertain chaotic systems, while in [14], an adaptive controller was designed based on Lyapunov stability theory for a class of chaotic systems with time-varying, unknown and bounded parameters, which led to the global asymptotic tracking of the desired bounded trajectories. Furthermore, in [15,16], a static output feedback approach was used to stabilize T–S fuzzy systems with time-varying norm bounded uncertainties, while in [17], an input–output approach based on data obtained from the underlying dynamical system was developed for modeling the adaptive control of an unknown chaotic system. Finally, in [18], the impulsive control scheme of as uncertain Luré system was also presented. However, there has been very little research done on the stability and design of impulsive fuzzy-model-based controllers for complex nonlinear systems with parameter uncertainties so far.

We intend in this paper to study impulsive fuzzy T–S models of chaotic systems with parameter uncertainties. Our main goal is to develop a unified and intelligent methodology aimed at obtaining various robust impulsive controllers targeting chaotic systems with uncertainties. In other words, we intend to combine impulsive control techniques with intelligent fuzzy logic methodology, so that more powerful controllers that can handle both chaos and parameter uncertainties can be generated. These newly designed controllers will possess four distinctive features: (1) a class of closed-loop chaotic systems was robustly stabilized for all admissible uncertainties; (2) the robust stability conditions obtained require only slight modifications to obtain asymptotic and exponential stability. Even their form remains almost the same; (3) several stability results are expressed in terms of unified linear matrix inequalities to effectively resolve the issue of control parameters; (4) the impulsive control of fuzzy models is very simple because it requires only the adjustment of impulsive distances, leading to the stabilization of various chaotic systems using small control impulses.

This paper is organized as follows. In Section 2, we develop an alternative representation for a class of chaotic systems through introducing impulses into uncertain T–S fuzzy models to generate a set of uncertain fuzzy impulsive T–S linear systems. By using this representation scheme, many advanced impulsive-based analysis and design techniques can be slightly modified and employed for constructing impulsive fuzzy-model-based controllers. In Section 3, we derive sufficient conditions for the synthesis of these controllers by using impulsive control approaches as well as LMI techniques combined with the application of a convex optimization technique. We also propose in this section a simple iterative algorithm for calculating the design parameters based on LMI techniques. Several numerical simulations are carried out in Section 4 to illustrate the proposed method.

2. Preliminaries

The aim of this work is to develop a robust control law for a class of chaotic systems with parameter uncertainties based on the Takagi–Sugeno (T–S) fuzzy model with impulse effects. In this type of fuzzy model, local dynamics in different state space regions are represented by linear system models. The overall model of the system is achieved by fuzzy "blending" of these linear models. The T–S fuzzy model can express a highly nonlinear functional relation with a small number of rules. In particular, we are interested in obtaining an exact and unified representation of many chaotic systems in a compact set of state variables, such as Lorenz, Rössler, Chua, Chen and Lü systems, etc.

Consider a class of chaotic systems, which can be exactly represented in the following unified T–S fuzzy model.

$$\begin{aligned}&\text{Rule } i: \text{IF } z_1(t) \text{ is } M_{i1}, \text{ and } z_2(t) \text{ is } M_{i2}, \cdots\cdots, \text{ and } z_n(t) \text{ is } M_{in},\\&\text{THEN } \dot{x}(t) = A_i x(t) \quad i = 1, 2, \cdots, r.\end{aligned} \tag{1}$$

where M_{ij} is a fuzzy set, $z(t) = [z_1(t), z_2(t), \cdots, z_n(t)]^T$ is a premise variable, $x(t) \in R^n$ is a state vector, $A_i \in R^{n\times n}$ are system matrices, r is fuzzy rule number.

So far, many chaos controllers have been developed through adding an extra input $u(t)$ in each subsystem of (1), that is, $\dot{x}(t) = A_i x(t) + u(t)$ [6,14–17]. Recently, a new T–S model with impulse effects has been proposed for chaos control, where each impulse is viewed as a controller to be designed [10,11]. In this controller, given a set of control instants $T = \{\tau_k\}$, $\tau_k \in R$, $\tau_k < \tau_{k+1}$, $k = N$, where N is the set of natural numbers. According to our impulsive control strategy, we only need to modify the changeable state variables at discrete instants called control instants. That is, at each τ_k, the state variable $x(t)$ is altered instantaneously by $x(\tau_k^+) = x(\tau_k^-) + B_i x(t)$, which denotes the "jump" of the state variable at the instant τ_k, where $B_i \in R^{n\times n}$, $x(\tau_k^+)$ is the right limit of $x(t)$ at $t = \tau_k$, and $x(\tau_k^-)$ is the left limit of $x(t)$, in other words, $x(\tau_k^+) = \lim\limits_{t\to\tau_k^+} x(t)$, $x(\tau_k^-) = \lim\limits_{t\to\tau_k^-} x(t)$. Based this idea, the impulsive T–S fuzzy of (1) is written as:

$$\begin{aligned}&\text{Rule } i: \text{IF } z_1(t) \text{ is } M_{i1}, \text{ and } z_2(t) \text{ is } M_{i2}, \cdots\cdots, \text{ and } z_n(t) \text{ is } M_{in},\\&\text{THEN } \begin{cases}\dot{x}(t) = A_i x(t), & t \neq \tau_k\\ \Delta x(\tau_k) = B_i x(t), & t = \tau_k\end{cases} \quad i = 1, 2, \cdots, r, \quad k = 1, 2, \cdots\end{aligned} \tag{2}$$

where $\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-)$.

In practice, however, some parameters of chaotic systems cannot be exactly known in priori and uncertainties always appear in controllers for many reasons, including finite word length in digital systems, the imprecision inherent in analogy systems, and the need for additional tuning of parameters in the final controller implementation [13–19]. Thus, it is very interesting to study the design of robust impulsive fuzzy controllers with respect to parameter perturbations. A class of impulsive T–S chaotic systems with the parameter uncertainties can be given by

$$\begin{aligned}&\text{Rule } i: \text{IF } z_1(t) \text{ is } M_{i1}, \text{ and } z_2(t) \text{ is } M_{i2}, \cdots\cdots, \text{ and } z_n(t) \text{ is } M_{in},\\ &\text{THEN } \begin{cases}\dot{x}(t) = (A_i + \nabla A_i)x(t), & t \neq \tau_k\\ \Delta x(\tau_k) = (B_i + \nabla B_i)x(t), & t = \tau_k\end{cases} \quad i = 1, 2, \cdots, r, \quad k = 1, 2, \cdots\end{aligned} \tag{3}$$

where ∇A_i and ∇B_i are unknown matrices representing the parameter perturbations. In [19], the nonlinear control of a three-cylinder spark ignition engine with parameter uncertainties was discussed based on the T–S fuzzy model and robust control approach. In this paper, we use the methods of [19] for representing parameter uncertainties, that is, the parameter uncertainties are classically written as $\nabla A_i = D_i F_i E_i$, $\nabla B_i = \overline{D}_i \overline{F}_i \overline{E}_i$, where D_i, E_i, $\overline{D}_i$, $\overline{E}_i$ are known real matrices of appropriate dimension, F_i, $\overline{F}_i$ are the unknown matrix functions with Lebesgue-measurable elements and satisfy the conditions:

$$F_i^T F_i \leqslant I, \quad \overline{F}_i^T \overline{F}_i \leqslant I,$$

where I is the identity matrix of appropriate dimension.

Using classical center-average defuzzification, product inference and singletone fuzzifier, the impulsive fuzzy chaos control system (3) with parameter uncertainties becomes

$$\begin{cases}\dot{x}(t) = \sum_{i=1}^{r}(A_i + D_i F_i E_i)x(t) & t \neq \tau_k\\ \Delta x(\tau_k) = \sum_{i=1}^{r} h_i(z(t))(B_i + \overline{D}_i \overline{F}_i \overline{E}_i)x(t) & t = \tau_k\end{cases} \tag{4}$$

where $h_i(z(t)) = \frac{\omega_i(z(t))}{\sum_{i=1}^{r}\omega_i(z(t))}$, $\omega_i(z(t)) = \prod_{j=1}^{n} M_{ij}(z_j(t))$, $i = 1, 2, \cdots, r$, and $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}. Thus

$$\omega_i(z(t)) \geqslant 0, \sum_{i=1}^{r}\omega_i(z(t)) > 0, i = 1, 2, \cdots, r, \quad h_i(z(t)) \geqslant 0, \quad \sum_{i=1}^{r} h_i(z(t)) = 1, \quad i = 1, 2, \cdots, r.$$

Beside the uncertainty of chaotic systems, we discuss in this paper several stability criteria (such as robust stability, asymptotic stability, and exponential stability) at the same time. We focus our attention on the designs of several robust controllers in the form of system (4) such that

(S1) A class of closed-loop chaotic systems is robustly stabilized for all admissible uncertainties.
(S2) The robust stability conditions can only require slight revisions to achieve asymptotic and exponential stabilities of chaotic systems in such away that their forms remain almost the same.
(S3) Several stability results are expressed in terms of unified linear matrix inequalities to effectively resolve the issue of control parameters.
(S4) The control method requires only the adjustment of impulsive distances, leading to the stabilization of various chaotic systems using small control impulses.

Before concluding this section, we recall certain matrix inequalities that will be used in the next section.

Lemma 1. ([22]). *Let A, E, F and Σ be real matrices of appropriate dimensions, with Σ satisfying $\|\Sigma\| \leqslant 1$. Then we have*

(a) *For any scalar $\lambda > 0$,*

$$E\Sigma F + E^T\Sigma^T F^T \leqslant \lambda^{-1}EE^T + \lambda FF^T.$$

(b) *For any matrix $P > 0$ and scalar $\xi > 0$ such that $\xi I - F^T P F > 0$,*

$$(A + E\Sigma F)^T P(A + E\Sigma F) \leqslant A^T PA + A^T PF(\xi I - F^T PF)^{-1} F^T PA + \xi^{-1} E^T E.$$

3. Main results and design algorithm

In this section, we present our main results on the robust stabilization of system (4), and design an iterative algorithm for calculating control parameters.

Theorem 1. *Assume that there exist three positive constants $\lambda > 0, \xi > 0, \beta > 0, \gamma > 0$ and $\theta > 0$, and positive definite matrix P, such that*

$$(i)\quad \begin{bmatrix} PA_i + A_i^T P - \theta P & E_i & PD_i \\ E_i^T & -\lambda I & 0 \\ D_i^T P & 0 & -\lambda^{-1} I \end{bmatrix} \leqslant 0, \quad i = 1, 2, \cdots, r,$$

$$(ii)\quad \begin{bmatrix} (I+B_i)^T P(I+B_i) - \beta P_i & \overline{D}_i & \overline{E}_i^T P(I+B_i) \\ \overline{D}_i^T & -\xi^{-1} I & 0 \\ (I+B_i)^T P\overline{E}_i & 0 & -(\xi I - \overline{E}_i^T P \overline{E}_i) \end{bmatrix} \leqslant 0, \quad i = 1, 2, \cdots, r.$$

Then the trivial solution of the impulsive fuzzy system (4) is stable if $\theta(\tau_{k+1} - \tau_k) + \ln(\beta) \leqslant 0$; asymptotically stable if $\theta(\tau_{k+1} - \tau_k) + \ln(\eta\beta) \leqslant 0$; and exponentially stable if $\ln(\beta) < -(\theta + \gamma)d$, where $k \in N$ and $0 < \tau_{k+1} - \tau_k \leqslant d$.

Proof. Consider the following Lyapunov function $V(t,x) = x^T P x$. For $t \neq t_k$, the time derivative of $V(t, x(t))$ along a given solution trajectory is given by

$$D^+V(t,x(t)) = \dot{x}^T P x + x^T P \dot{x} = \sum_{i=1}^{r} h_i(z(t)) x^T[(A_i^T P + PA_i) + 2PD_iF_iE_i]x \leqslant \sum_{i=1}^{r} h_i(z(t)) x^T(A_i^T P + PA_i + \lambda^{-1}PD_iD_i^T P^T + \lambda E_i^T E_i)x$$

Employing Schur criterion to condition (i), we have

$$PA_i + A_i^T P + \lambda^{-1} PD_iD_i^T P + \lambda E_i^T E_i \leqslant \theta P.$$

This implies that

$$D^+V(t,x(t)) \leqslant \theta x^T P x = \theta V(t,x). \tag{5}$$

For $t = t_k$, we have

$$\begin{aligned} V(t_k^+, x(t_k^+)) &= \sum_{i=1}^{r} h_i(z(t_k) x_k^T [(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]^T P \sum_{j=1}^{r} h_j(z(t_k)[(I+B_j) + \overline{D}_j\overline{F}_j\overline{E}_j] x_k \\ &= \sum_{i=1}^{r} h_i^2(z(t_k) x_k^T [(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]^T P[(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i] x_k \\ &\quad + \sum_{1 \leqslant i<j \leqslant r} h_i(z(t_k)h_j(z(t_k)x_k^T\{[(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]^T P[(I+B_j) + \overline{D}_j\overline{F}_j\overline{E}_j] \\ &\quad + [(I+B_j) + \overline{D}_j\overline{F}_j\overline{E}_j]^T P(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]\} x_k \\ &\leqslant \sum_{i=1}^{r} h_i^2(z(t_k) x_k^T [(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]^T P[(I+B_i) \\ &\quad + \overline{D}_i\overline{F}_i\overline{E}_i] x_k + \sum_{1 \leqslant i<j \leqslant r} h_i(z(t_k)h_j(z(t_k)x_k^T\{[(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]^T P(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i] \\ &\quad + [(I+B_j) + \overline{D}_j\overline{F}_j\overline{E}_j]^T P[(I+B_j) + \overline{D}_j\overline{F}_j\overline{E}_j]\} x_k \\ &= \sum_{i=1}^{r} h_i(z(t_k) x_k^T [(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i]^T P[(I+B_i) + \overline{D}_i\overline{F}_i\overline{E}_i] x_k \end{aligned} \tag{6}$$

Applying Lemma 1(b) on the right-hand side of (6), we obtain

$$V(t_k^+, x(t_k^+)) \leqslant \sum_{i=1}^{r} h_i(z(t_k) x_k^T [(I+B_i)^T P(I+B_i) + (I+B_i)^T P\overline{E}_i(\xi I - \overline{E}_i^T P\overline{E}_i)^{-1}\overline{E}_i^T P(I+B_i) + \xi^{-1}\overline{D}_i^T\overline{D}_i] x_k$$

Similarly, by applying Schur criterion on condition (ii), we get

$$(I+B_i)^T P(I+B_i) + (I+B_i)^T P\overline{E}_i(\xi I - \overline{E}_i^T P\overline{E}_i)^{-1}\overline{E}_i^T P(I+B_i) + \xi^{-1}\overline{D}_i^T\overline{D}_i \leqslant \beta P.$$

In other words,

$$V(t_k^+, x(t_k^+)) \leqslant \beta V(t_k, x_k) \tag{7}$$

Let $x(t) \equiv x(t, t_0, x_0)$ be any solution of (4) satisfying $\|x(t_0)\| < \delta$. It follows that, for any $\varepsilon \in (0, 1]$, we may choose $\delta = \delta(\varepsilon) > 0$ such that $c_2\delta^q < c_1\varepsilon^q e^{-(\alpha+c)m}$, i.e.,

$$V(t_0, x) \leqslant (c_1)^{1/q}\varepsilon. \tag{8}$$

For any $t \in (t_0, t_1]$, (4) implies

$$V(t,x) \leqslant V(t_0,x)\exp(\theta(t-t_0)).$$

Using the above inequality, we get

$$V(t_1,x) \leqslant V(t_0,x)\exp(\theta(t_1-t_0)) \tag{9}$$

Inequalities (6) and (8) imply

$$V(t_1^+, x(t_1^+)) \leqslant \beta V(t_1,x) \leqslant \beta V(t_0,x)\exp(\theta(t_1-t_0)).$$

Similarly, for $t \in (t_1, t_2]$, we have

$$\begin{aligned} V(t,x) &\leqslant V(t_1^+,x)\exp(\theta(t-t_1)) \leqslant \beta V(t_0,x)\exp(\theta(t-t_0))V(t_2^+,x(t_2^+)) \leqslant \beta V(t_2,x) \leqslant \beta V(t_1^+,x)\exp(\theta(t_2-t_1)) \\ &\leqslant \beta^2 V(t_0,x)\exp(\theta(t_1-t_0))\exp(\theta(t_2-t_1)) \leqslant \beta^2 V(t_0,x)\exp(\theta(t-t_0)) \end{aligned}$$

By employing mathematical induction, we get for any $t \in (t_{j-1}, t_j]$,

$$V(t,x) \leqslant \beta^j V(t_0,x)\exp(\theta(t-t_0)). \tag{10}$$

According to (10), there are three cases to consider: (I) In view of conditions $\theta(\tau_j - \tau_{j-1}) + \ln(\beta) \leqslant 0$ and (10), we have

$$\begin{aligned} V(t,x) &\leqslant \beta^j V(t_0,x)\exp(\theta(t-t_0)) \leqslant V(t_0,x)\exp(-\theta(t_1-t_0))\exp(-\theta(t_2-t_1))\cdots\exp(-\theta(t_j-t_{j-1}))\exp(\theta(t-t_0)) \\ &\leqslant V(t_0,x)\exp(-\theta(t_j-t_0))\exp(\theta(t-t_0)) = V(t_0,x)\exp(\theta(t-t_j)) \leqslant V(t_0,x)\exp(\theta d) \end{aligned}$$

By using (8), the above implies

$$\|x(t)\| \leqslant \left(\frac{1}{c_1}\right)^{1/2} V(t_0,x)\exp(\theta d/2) \leqslant \varepsilon \exp(\theta d/2). \tag{11}$$

Since ε, θ, d are constants, it follows that (4) is stable. (II) Similar to (11), when $d_j \leqslant \frac{1}{\eta}\exp(-\alpha(\tau_j - \tau_{j-1}))$, we have

$$V(t,x) \leqslant \beta^j V(t_0,x)\exp(\theta(t-t_0)) \leqslant \left(\frac{1}{\eta}\right)^j V(t_0,x)\exp(\theta d)$$

and

$$\|x(t)\| \leqslant \left(\frac{1}{c_1}\right)^{1/2}\left(\frac{1}{\eta}\right)^{j/2} V(t_0,x)\exp(\theta d/2) \leqslant \varepsilon\left(\frac{1}{\eta}\right)^{j/2}\exp(\theta d/2). \tag{12}$$

Since $\eta > 1$, it follows that (4) is asymptotically stable when $j \to \infty$. (III) When $\ln(\beta) < -(\theta+\gamma)d$, we have

$$\begin{aligned} V(t,x) &\leqslant \beta^j V(t_0,x)\exp(\theta(t-t_0)) \leqslant \overbrace{\exp(-(\theta+\gamma)d))\cdots\exp(-(\theta+\gamma)d))}^{j} V(t_0,x)\exp(\theta(t-t_0)) \\ &= \exp(-j\gamma d))\exp(-j\theta d)V(t_0,x)\exp(\theta(t-t_0)) \\ &\leqslant \exp(-j\gamma d))\exp(-\theta[(t_1-t_0)+(t_1-t_0)+\cdots+(t_j-t_{j-1})])V(t_0,x)\exp(\theta(t-t_0)) \\ &\leqslant V(t_0,x)\exp(-j\gamma d))\exp(-\theta(t_j-t_0))\exp(\theta(t-t_0)) = V(t_0,x)\exp(-j\gamma d))\exp(\theta(t-t_j) \\ &\leqslant V(t_0,x)\exp(-j\gamma m))\exp(\theta d) \end{aligned}$$

and

$$\|x(t)\| \leqslant \left(\frac{1}{c_1}\right)^{1/2} V(t_0,x)\exp(-j\gamma d/2))\exp(\theta d/2) \leqslant \varepsilon\exp(-j\gamma d/2))\exp(\theta d/2) \tag{13}$$

which implies that (4) is exponentially stable. □

Remark 1. Theorem 1 states that $0 < \beta < 1$ and $\|I + B_i\| < 1$. Hence we may choose the matrices B_i to satisfy $\|I + B_i\| < 1$, and then calculate P by using an iterative method. Thus conditions (i) and (ii) of Theorem 1 are linear matrix inequalities with respect to P.

In the following, we present a design algorithm for calculating the control parameters in Theorem 1.

Algorithm 1. In order to reduce control or implementation cost of chaotic systems, it would be desirable to choose impulsive distances as large as possible. Hence, according to Theorem 1, we have to choose θ and $0 < \beta < 1$ as small as possible to guarantee large impulsive distances. Meanwhile, we may choose values for $\gamma, \lambda, \xi > 0$ and matrices B_i such that $\|I + B_i\| < 1$. To summarize, the five steps required to stabilize system (3) are

(1) Set a threshold T, initialize $\theta > 0$, $\beta > 0$ (e.g., by setting $\theta = 5$, $\beta = 0.1$);
(2) Calculate P by (i) and (ii) of Theorem 1;
(3) Stop if P exists. Otherwise set $\beta = \beta + \Delta\beta$;
(4) Repeat from step 2 if $\beta < 1$. Otherwise set $\theta = \theta + \Delta\theta$ and β equal its initial value;
(5) Repeat from step 2 if $\theta < T$. Otherwise fail.

Once the algorithm succeeds, we may determine a bound on the impulsive distances by utilizing the inequality $(\tau_{k+1} - \tau_k) \leqslant -\ln(\beta)/\theta$. For obtain asymptotical stability, however, Algorithm 1 can be slightly modified by letting $0 < \beta < 1/\eta$, since $(\tau_{k+1} - \tau_k) \leqslant -\ln(\eta\beta)/\theta$ and $\eta > 1$. Similarly, by letting $d < -\ln(\beta)/(\theta + \gamma)$ in Algorithm 1, the control parameters for exponential stability can be calculated.

4. Numerical simulations and discussions

In order to illustrate our results, the design algorithm for calculating the control parameters in Theorem 1 is applied on the Lorenz system and Chua's oscillator with parameter uncertainties. Before the simulations, we give here the general three-stage procedure for realizing chaos control. (1) We express the control structures of chaotic systems in the form of system (3), and set the corresponding parameters including those uncertain ones. In this paper, the elements of ∇A_i and ∇B_i are randomly chosen within 30% of their nominal values corresponding to B_{iB} and B_{iB}. (2) We apply the iterative algorithm proposed here to calculate the control parameters in Theorem 1, which includes P, β, θ and d, etc.. According to these parameters, we can determine an upper bound on impulse jump distances. (3) We use the Runge-Kutta algorithm for solving impulsive differential Eq. (4). In the implementation of the Runge–Kutta algorithm, we must add a condition to select and judgment sentence for controlling the impulse jump. This is jointly determined by impulse jump distance and the time step of the Runge–Kutta algorithm. In our examples, the integration step-size of the Runge–Kutta algorithm is set at [0,1] and the time step is chosen to be 0.0001. In the following, we will give two examples for illustrating the effectiveness of our methods.

Example 1. Lorenz system [20]

$$\begin{cases} \dot{x}_1(t) = a(x_2(t) - x_1(t)) \\ \dot{x}_2(t) = bx_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) = -cx_3(t) + x_1(t)x_2(t) \end{cases}.$$

where $x_1(t), x_2(t), x_3(t)$ are the state variables, a, b, c are the system parameters. The Lorenz system is chaotic if $a = 10$, $b = 28$, $c = 8/3$. We thus have the following impulsive fuzzy control model with parameter uncertainties.

Plant Rule i: IF $x_1(t)$ is M_i.

$$\text{THEN} \begin{cases} \dot{x}(t) = (A_i + \nabla A_i)x(t) & t \neq \tau_k \\ \Delta x|_{t=\tau_k} = x(t_k^+) - x(t_k^-) \equiv (B_i + \nabla B_i)x(t) & t = \tau_k \quad i = 1,2. \quad k = 1,2,\cdots \\ x(t_0^+) = x_0 \end{cases}$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$

$$A_1 = \begin{bmatrix} -a & a & 0 \\ b & -1 & -d \\ 0 & d & c \end{bmatrix}, \quad A_2 = \begin{bmatrix} -a & a & 0 \\ b & -1 & d \\ 0 & -d & c \end{bmatrix},$$

$B_1 = diag([d_{11}, d_{12}, d_{13}]), d_{21} = diag([d_{21}, d_{22}, d_{23}]), M_1(x_1(t)) = \frac{1}{2}(1 + \frac{x_1(t)}{d}),\ M_2(x_1(t)) = \frac{1}{2}(1 - \frac{x_2(t)}{d})$. The parameters a, b, c, d_{11}, $d_{12}, d_{13}, d_{21}, d_{22}$ and d_{23} are the system's parameters, while ∇A_i and ∇B_i represent the bounded parameter uncertainties. The elements of ∇A_i and ∇B_i are randomly chosen within 30% of their nominal values corresponding to A_i and B_i. Based on assumption of uncertainty, we define

$$D_i = \overline{D}_i = \text{diag}([0.3, 0.3, 0.3]), \quad E_i = \begin{bmatrix} -a & a & 0 \\ b & -1 & 0 \\ 0 & 0 & c \end{bmatrix}, \quad \overline{E}_i = B_i (i = 1, 2).$$

In this example, we choose $a = 10, b = 28, c = 8/3, B_1 = \text{diag}([-0.8, -0.5, -0.7]), B_2 = diag([-0.6, -0.5, -0.6])$, and d=30. In terms of Algorithm 1, we choose initial iteration values for θ and β as 5 and 0.1, respectively. Their increments, $\Delta\theta$ and $\Delta\beta$, are chosen to be 5 and 0.1. In this case, P=[1.1591, -0.1671, -0.0000; -0.1671,1.6621,-0.0000; -0.0000, -0.0000, 1.2307], $\theta = 500$ and $\beta = 0.5$. In other words, to achieve stability, the impulsive distances must satisfy $(\tau_{k+1} - \tau_k) \leqslant -\ln(\beta)/\theta = 0.0014$.

Example 2. Chua's oscillator [21]

$$\begin{cases} \dot{x}_1(t) = a(-x_1(t) + x_2(t) - f(x_1(t))) \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t), \quad f(x_1(t)) = m_1 x_1(t) + m_2(|x_1(t) + 1| - |x_1(t) - 1|), \\ \dot{x}_3(t) = bx_2(t) \end{cases}$$

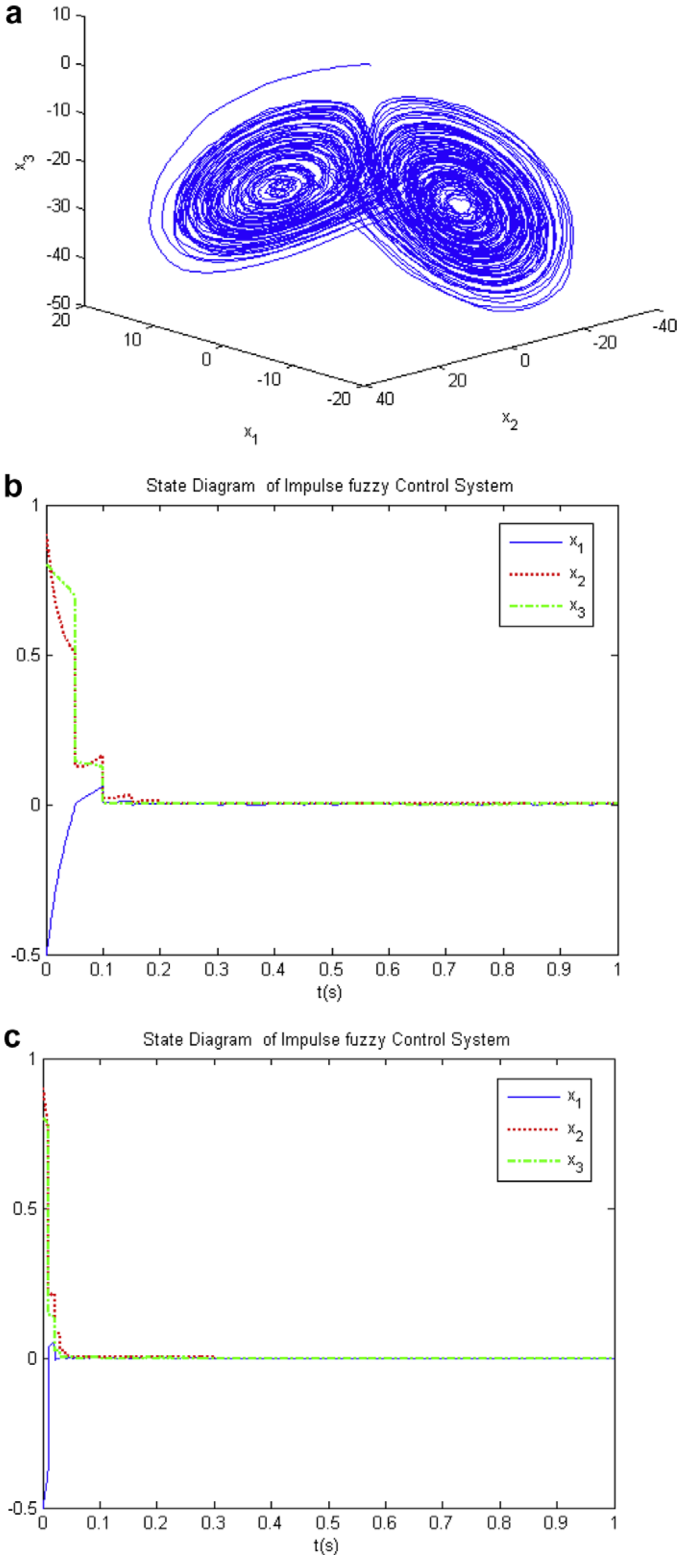

Fig. 1. (a) is phase diagram of uncertain Lorenz's system without control; (b) and (c) are state diagram of uncertain Lorenz's system under impulsive control for impulsive distances 0.0014 and 0.001, respectively.

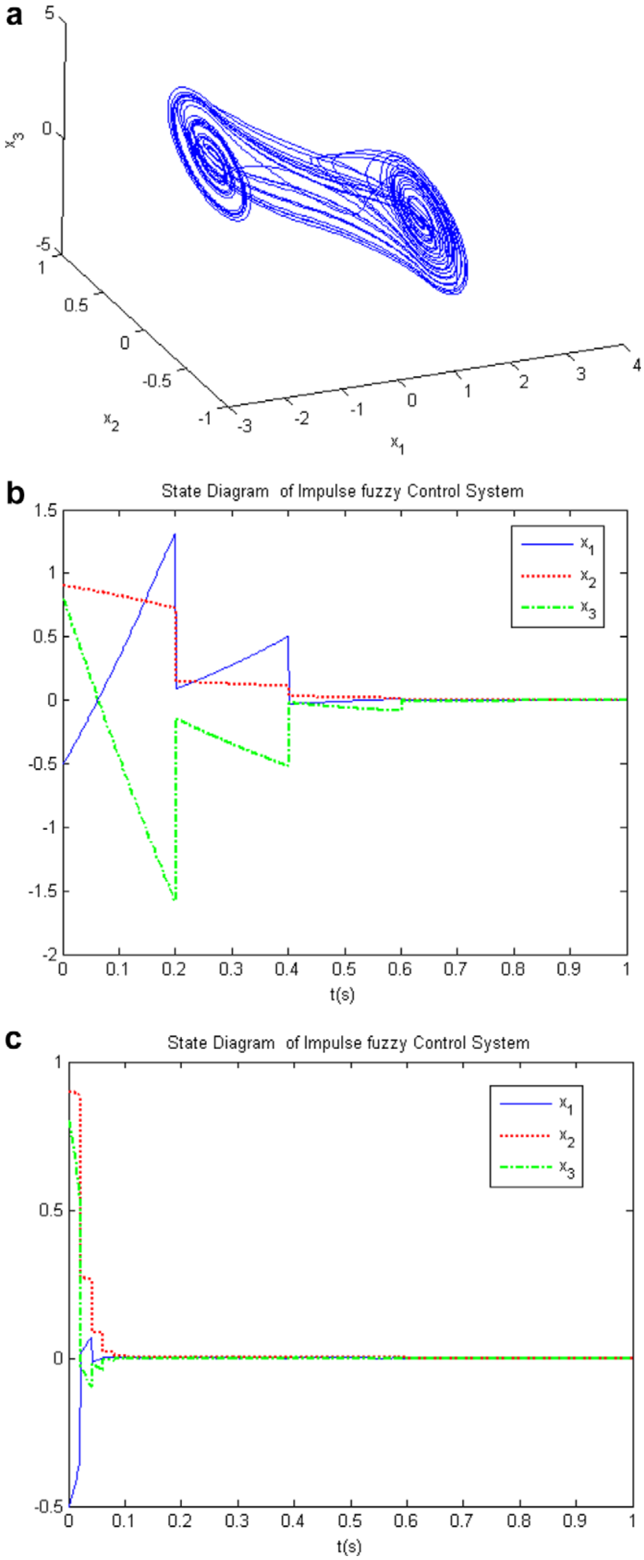

Fig. 2. (a) is phase diagram of uncertain Chua's system without control; (b) and (c) are state diagram of uncertain Chua's system under impulsive control for impulsive distances 0.0027 and 0.002, respectively.

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where $x_1(t), x_2(t), x_3(t)$ are the state variables, a, b, m_1, m_2 are the system parameters. The Chua's system is chaotic if $a = 10$, $b = 14.87$, $m_1 = -0.68$, $m_2 = -0.295$. We thus have the following impulsive fuzzy control model with parameter uncertainties.

Plant Rule i: IF $x_1(t)$ is M_i.

$$\text{THEN}\begin{cases}\dot{x}(t) = (A_i + \nabla A_i)x(t) & t \neq \tau_k \\ \Delta x|_{t=\tau_k} = x(t_k^+) - x(t_k^-) \equiv (B_i + \nabla B_i)x(t) & t = \tau_k \qquad i = 1,2. \quad k = 1,2,\cdots \\ x(t_0^+) = x_0 \end{cases}$$

where, $x(t) = [x_1(t), x_2(t), x_3(t)]^T$,

$$A_1 = \begin{bmatrix} a(d-1) & a & 0 \\ 1 & -1 & 1 \\ 0 & b & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -a(d+1) & a & 0 \\ 1 & -1 & 1 \\ 0 & b & 0 \end{bmatrix}, \quad \phi(x_1(t)) = \begin{cases} f(x_1(t))/x_1(t) & x_1(t) \neq 0 \\ -0.27 & x_1(t) = 0 \end{cases},$$

$M_1(x_1(t)) = 0.5(1 - \phi(x_1(t))/d), M_2(x_1(t)) = 1 - M_1(x_1(t)), B_1 = \mathrm{diag}([d11, d12, d13])$, and $B_2 = \mathrm{diag}([d21, d22, d23])$. The uncertain matrices ∇A_i and ∇B_i are defined as in Example 1, while

$$D_i = \overline{D}_i = \mathrm{diag}([0.3, 0.3, 0.3]), \quad E_i = \begin{bmatrix} -a & a & 0 \\ 0 & 0 & 0 \\ 0 & b & 0 \end{bmatrix}, \quad \overline{E}_i = B_i (i = 1, 2).$$

In this example, we let $a = 10$, $b = -14.87$, $B_1 = \mathrm{diag}([-0.8, -0.5, -0.7])$, $B_2 = diag([-0.6, -0.5, -0.6])$, and $d = 3$. In terms of Algorithm 1, we choose the initial iteration values of θ and β to be 5 and 0.1, respectively, and their increments, $\Delta\theta$ and $\Delta\beta$ to be 5 and 0.1. This implies that P=[1.2596 -0.0263 -0.1959;-0.0263 1.2993 -0.0278; -0.1959 -0.0278 1.3872], and $\theta = 260$ and $\beta = 0.5$. It follows that the Chua's oscillator is stable whenever $(\tau_{k+1} - \tau_k) \leqslant -\ln(\beta)/\theta = 0.0027$.

Based on the above discussions, we are now able to show simulation results of both examples. Fig. 1a shows the phase diagram of an uncertain Lorenz system without the impulses; (b) and (c), on the other hand, show the time series of an uncertain Lorenz exposed to a sequence of equidistant impulses with distances of 0.0014 and 0.001, respectively. Similarly, Fig. 2 shows the same type of simulations for the uncertain Chua's oscillator with impulsive distances of 0.0027 and 0.002, respectively.

These simulations reveal that smaller impulsive distances imply faster convergence to the zero equilibrium. They also show that the impulsive control of fuzzy models is very simple because it requires only the adjustment of impulsive distances, leading to the stabilization of various chaotic systems using small control impulses.

5. Conclusions

In this paper, a proposed mechanism to combine impulsive fuzzy models with chaos control to generate a class of fuzzy chaotic systems with parameter uncertainties, have been investigated. Several unified criteria for stability, asymptotic stability and exponential stability have been presented in the form of LMI so that robust controllers can be solved efficiently by using programming techniques. In addition, we have also designed an iterative algorithm for calculating control parameters under various stability conditions based on LMI techniques. We demonstrated how effective this algorithm is by showing that it is feasible to impulsively stabilize T–S fuzzy models based on chaotic systems by using only small control impulses.

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