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Impulsive stability of chaotic systems represented by T-S model

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ABSTRACT

In this paper, a novel and unified control approach that combines intelligent fuzzy logic methodology with impulsive control is developed for controlling a class of chaotic systems. We first introduce impulses into each subsystem of T-S fuzzy IF-THEN rules and then present a unified T-S impulsive fuzzy model for chaos control. Based on the new model, a simple and unified set of conditions for controlling chaotic systems is derived by Lyapunov method techniques and a design procedure for estimating bounds on control matrices is also given. These results are shown to be less conservative than those existing ones in the literature and several numerical examples are presented to illustrate the effectiveness of this method.

1. Introduction

In recent years, it has become quite popular to adopt the so-called Takagi–Sugeno (T-S) type fuzzy models [1–7,17–20] to represent or approximate a nonlinear system. These fuzzy models are described by a family of fuzzy IF-THEN rules which represent local linear input–output relations of a nonlinear system. The overall fuzzy models are achieved by smoothly blending these local linear models together through fuzzy membership functions. As a result, it has been feasible to apply conventional linear system theory to analyze nonlinear control systems [1,3,8]. In particular, the stability issue of chaotic systems represented by T-S fuzzy model has been investigated extensively in a unified way [2,3,5,8]. Under the framework of T-S fuzzy model, we found that the existing controllers of the chaotic system represented by T-S model had almost employed PDC and LMI techniques [2,3,5,8], where PDC was used for converting T-S fuzzy rules into closed-loop systems while LMI was employed to find common positive definite matrices. However, PDC is a continuous input control method and not available for the development of digital control devices. On the other hand, impulsive control is an attractive alternative because it not only allows the stabilization of a nonlinear system using only small control impulses but also offers a direct method for modulating digital information onto a chaotic carrier signal for spread spectrum applications. Moreover, impulsive control may also offer a simple and efficient method to deal with systems based on the development of digital control devices which generate control impulses at discrete moments [9–16]. But the existing impulsive control theory lacks the unified way for dealing with different chaotic systems. Therefore, it is necessary to develop a new technique for controlling chaos in such a way that impulsive control characterizes fuzzy intelligence. It is important to point out that combining impulsive control with fuzzy modeling for chaotic system has been rarely discussed until now.

This paper is devoted to providing an alternative and novel approach for controlling T-S fuzzy chaotic systems based on impulsive control techniques. That is, under the framework of T-S fuzzy rules, impulses are introduced into every subsystem of T-S fuzzy chaotic systems where these impulses are viewed as the corresponding control inputs that need to be designed. In this case, the overall fuzzy models become impulsive differential equations obtained by smoothly blending these local

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linear impulsive models together through fuzzy membership functions. In other words, the intelligent fuzzy logic methodology will be combined with simple impulsive control techniques so as to develop a more powerful controller that can efficiently stabilize complex chaotic systems by using only small impulses. It will be demonstrated that this novel control model not only provides a unified approach for controlling a class of chaotic systems but also possesses the control merits of impulsive control techniques.

The rest of this paper is organized as follows. Section 2 describes a fuzzy modeling methodology for T-S fuzzy chaotic systems and presents impulsive fuzzy control model for controlling complex chaotic systems. In Section 3, theoretical stability analysis of impulsive fuzzy chaotic systems is developed, while, in Section 4, several numerical simulations of chaotic systems are carried out based on the proposed method. Finally, in Section 5, some concluding remarks are made.

2. Impulsive control models for fuzzy chaotic systems

It is well known that many chaotic systems, such as Lorenz, Chen's, Chua's and Rossler systems, can be exactly represented by a unified fuzzy T-S model proposed by Takagi and Sugeno [2]. In order to emphasize the advantages of our methods over the conventional T-S model, we initially consider the case when the ith rules of the T-S fuzzy models for the chaotic systems are of the following forms [1,3]:

Plant Rule i: IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}

$$\text{THEN } \dot{x}(t) = A_i x(t), \quad i = 1, 2, \ldots, r, \tag{1}$$

where M_{ij} is the fuzzy set and r is the number of IF-THEN rules, $x(t) \in R^m$ is the state vector, $A_i \in R^{n\times m}$ are known constant matrices, $z_1(t),\ldots,z_p(t)$ are the premise variables.

Remark 1. Under the framework of (1), the various approaches to controlling chaos have been investigated based on LMI and PDC techniques. Up to now, we found that the existing methods for controlling chaos based on fuzzy T-S model have solely used PDC technique, in which LMI algorithm has been used for finding common symmetry positive matrices. It should be pointed out, however, that the existing methods belong to continuous input control.

From the above discussion, we shall introduce a discrete input control into system (1) and propose a novel model so as to extend ordinary T-S model. To construct an impulsive or discrete control plant from a fuzzy T-S model employing a chaotic system, consider first a discrete set $\{\tau_j\}$ of time instants, where $0 < \tau_1 < \tau_2 < \cdots < \tau_j < \tau_{j+1} < \cdots$, $\tau_j \to \infty$, as $j \to \infty$. Let $\Delta x|_{\tau=\tau_j} \equiv x(\tau_j^+) - x(\tau_j^-)$ be the "jump" in the state variable at the time instant τ_j, where $x(\tau_j^-) = x(\tau_j)$. The impulsive control structure of the nonlinear systems represented by T-S fuzzy model is then defined by

Plant Rule i: IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip}

$$\text{THEN}\begin{cases} \dot{x}(t) = A_i x(t) & t \neq \tau_j, \\ \Delta x = K_{i,j} x(t) & t = \tau_j, \end{cases} \quad i = 1, 2, \ldots, r, \quad j = 1, 2, \ldots \tag{2}$$

where M_{ij}, $A_i \in R^{n\times m}$, $x(t) \in R^m$, $z_1(t),\ldots,z_p(t)$ and r have the same meanings as in (1), and $K_{i,j} \in R^{n\times m}$ denotes the control gain of the jth impulsive instant τ_j.

The defuzzified output of the impulsive T-S fuzzy system is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) A_i x(t) & t \neq \tau_j, \\ \Delta x = \sum_{i=1}^{r} h_i(z(t)) K_{i,j} x & t = \tau_j, \end{cases} \tag{3}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_p(t)]^{\mathrm{T}}$, $w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t))$, $\sum_{i=1}^{r} w_i(z(t)) > 0, w_i(z(t)) \geqslant 0$, $h_i(z(t)) = w_i(z(t))/\sum_{i=1}^{r} w_i(z(t))$, $\sum_{i=1}^{r} h_i(z(t)) = 1, h_i(z(t)) \geqslant 0$, $i = 1, 2, \ldots, r$. Note that system (3) will be called *impulsive fuzzy model* hereafter.

Remark 2. There exist some obvious differences between the local subsystems of (1) and (2). In fact, the former is a linear system, while the latter is an impulsive one. Therefore we tend to think of (2) as being an extension of (1).

Remark 3. The stability of system (3) can be analyzed by designing a set of control matrices $K_{i,j}$ and impulsive control intervals. Moreover, this unified impulsive control strategy is available for a class of chaotic systems due to the fact that (3) has the ability to unify a whole class of chaotic systems, such as Lorenz, Chen's and Chua's systems.

3. Impulsive stability of fuzzy chaotic systems

In this section, we shall focus our attention on the stability properties of impulsive fuzzy chaotic systems and derive sufficient conditions leading to the impulsive control of these systems.

Theorem 1. *Let P be an $n \times n$ symmetric and positive definite matrix and $\lambda_{max}(P)$ and $\lambda_{min}(P)$ be the largest and smallest eigenvalues of P, respectively. If there exist constant scalars $\kappa > 0\gamma > 0$, $\beta_j (j \in N)$, $\xi > 1$, such that*

$$(1)\quad PA_i + A_i^{\mathrm{T}}P - \kappa I < 0 \;\; (i = 1, 2, \ldots, r), \quad \gamma = \kappa/\lambda_{min}(P), \tag{4}$$

$$(2)\quad \beta_j = \frac{\lambda_{max}(P)}{\lambda_{min}(P)} \sum_{i=1}^{r} \|(I + K_{i,j})\|, \tag{5}$$

$$(3)\quad \ln(\xi\beta_j) + \gamma\sigma_j \leqslant 0, \tag{6}$$

where $0 < \sigma_j = \tau_j - \tau_{j-1} < \infty$ is the impulsive interval, then the impulsive fuzzy control system in Eq. (3) is globally asymptotically stable at the origin.

Proof. Consider a Lyapunov function of the form $V(x) = \frac{1}{2}x^{\mathrm{T}}Px$.

The time derivative of $V(x)$along the solution of Eq. (3) is

$$\dot{V}(x) = \frac{1}{2}x^{\mathrm{T}} \sum_{i=1}^{r} h_i(z(t))(PA_i + A_i^{\mathrm{T}}P)x \;\; t \in (t_{i-1}, t_i],$$

where $\lambda_i > 0$ for $h_i(z(t)) \in [0\ 1]$. By (4), we have

$$\begin{aligned} \dot{V}(x) &= \frac{1}{2}\sum_{i=1}^{r} h_i(z(t))x^{\mathrm{T}}[(PA_i + A_i^{\mathrm{T}}P)]x \leqslant \frac{1}{2}\kappa\|x\| \leqslant \frac{\kappa}{\lambda_{\min}(P)}V(x(t)). \\ &= \gamma V(x(t)) \;\; t \in (t_{j-1}, t_j] \quad (i = 1, 2, \ldots). \end{aligned} \tag{7}$$

On the other hand, it follows from the second equation in (3) and the properties of $h_i(z(t))$ that

$$\begin{aligned} V(x(\tau_j^+)) &= \frac{1}{2}\sum_{i=1}^{r} h_i(z(t))[(I + K_{i,j})x(\tau_j)]^{\mathrm{T}}P\sum_{i=1}^{r} h_i(z(t))(I + K_{i,j})x(\tau_j) \\ &\leqslant \frac{1}{2}\lambda_{\max}(P)\left[\sum_{i=1}^{r} h_i(z(t))(I + K_{i,j})^{\mathrm{T}}x^{\mathrm{T}}(\tau_j)\right]\left[\sum_{i=1}^{r} h_i(z(t))(I + K_{i,j})x(\tau_j)\right] \\ &= \frac{1}{2}\lambda_{\max}(P)\left\|\sum_{i=1}^{r} h_i(z(t))(I + K_{i,j})x(\tau_j)\right\| \\ &\leqslant \frac{1}{2}\lambda_{\max}(P)\sum_{i=1}^{r}\|(I + K_{i,j})\|\|x(\tau_j)\| \\ &\leqslant \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\sum_{i=1}^{r}\|(I + K_{i,j})\|V(x(\tau_j)) \\ &= \beta_j V(x(\tau_j)) \;\; j \in N \end{aligned} \tag{8}$$

By letting $j = 1$ in inequality (7), we obtain for any $t \in (t_0, t_1]$

$$V(x(t)) \leqslant V(x(t_0))\exp(\gamma(t - t_0)),$$

which implies that

$$V(x(t_1)) \leqslant V(x(t_0))\exp(\gamma(t_1 - t_0))$$

and $V(x(t_1^+)) \leqslant \beta_1 V(x(t_1)) \leqslant \beta_1 V(x(t_0))\exp(\gamma(t_1 - t_0))$.

Similarly for any $t \in (t_1, t_2]$

$$V(x(t)) \leqslant V(x(t_1^+))\exp(\gamma(t - t_1)) \leqslant \beta_1 V(x(t_0))\exp(\gamma(t - t_0)).$$

In general for any $t \in (t_j, t_{j+1}]$,

$$V(x(t)) \leqslant V(x(t_0))\beta_1\beta_2\cdots\beta_j\exp(\gamma(t - t_0)).$$

From inequality (6), we may conclude that

$$\beta_j\exp(\gamma\sigma_j) \leqslant 1/\xi, \quad j \in N.$$

Thus for $t \in (t_j, t_{j+1}]$, $j \in N$

$$\begin{aligned} V(x(t)) &\leqslant V(x(t_0))\beta_1\beta_2\cdots\beta_j\exp(\gamma(t - t_0)) = V(x(t_0))[\beta_1\exp(\gamma\sigma_1)]\cdots[\beta_j\exp(\gamma\sigma_j)]\exp(\gamma(t - t_j)) \\ &\leqslant V(x(t_0))\frac{1}{\xi^j}\exp(\gamma\sigma_{j+1}). \end{aligned}$$

Then it follows from the above inequality that the trivial solution of system (3) is globally asymptotically stable. □

In general, since A_i, $i = 1,2,\ldots,r$, of many chaotic systems are simple, we can directly employ the eigenvalues of the matrices to obtain the stability conditions for the impulsive fuzzy system.

Theorem 2. *Let P be an $n \times n$ symmetric and positive definite matrix whose largest and smallest eigenvalues are $\lambda_{max}(P)$ and $\lambda_{min}(P)$, respectively. Let λ_i be the largest eigenvalue of $PA_i + A_i^T P$ $(i = 1,2,\ldots,r)$, $\gamma = \max_{i=1,2,\ldots,r}\{\lambda_i\}/\lambda_{min}(P) > 0$, and $\beta_j = \frac{\lambda_{max}(P)}{\lambda_{min}(P)}\sum_{i=1}^{r}\|(I + K_{i,j})\|$ $(j = 1,2,\ldots)$. If there exists a constant $\xi > 1$ such that*

$$\ln(\xi\beta_j) + \gamma\sigma_j \leqslant 0 \quad (j = 1,2,\ldots), \tag{9}$$

where $0 < \sigma_j = \tau_j - \tau_{j-1} < \infty$ $(j \in N)$ is the impulsive interval, then system (3) is globally asymptotically stable at the origin.

Proof. Similar to the proof of Theorem 1. □

Theorem 1 can be simplified further by assuming $K = K_{i,j}$ and $\sigma = \sigma_j$ for all i, j, as illustrated in the following corollary.

Corollary 1. *Let P be an $n \times n$ symmetric and positive definite matrix and $\lambda_{max}(P)$ and $\lambda_{min}(P)$ be largest and smallest eigenvalues of P, respectively. If there exist constant scalars $\kappa > 0\gamma > 0$, β, $\xi > 1$, such that*

$$(1) \quad PA_i + A_i^T P - \kappa I < 0 \;\; (i = 1,2,\ldots,r), \quad \gamma = \kappa/\lambda_{min}(P), \tag{10}$$

$$(2) \quad \beta = \frac{\lambda_{max}(P)}{\lambda_{min}(P)}\|(I + K)\|, \tag{11}$$

$$(3) \quad \ln(\xi\beta) + \gamma\sigma \leqslant 0 \quad (j \in N), \tag{12}$$

where $0 < \sigma = \tau_j - \tau_{j-1} < \infty (j \in N)$ is the impulsive interval, then the impulsive fuzzy control system (3) is globally asymptotically stable at the origin.

Similarly, by assuming that $P = IK = K_{i,j}$ and $\sigma = \sigma_j$ for all i, j, in Theorem 2, we obtain the following corollary.

Corollary 2. *Let λ_i be the largest eigenvalue of $A_i + A_i^T$ $(i = 1,2,\ldots,r)$, $\gamma = \max_{i=1,2,\ldots,r}\{\lambda_i\} > 0$ and $\beta = \|(I + K)\| (j = 1,2,\ldots)$. If there exists a constant $\xi > 1$ such that*

$$\ln(\xi\beta) + \gamma\sigma \leqslant 0 \quad (j = 1,2,\ldots), \tag{13}$$

where $0 < \sigma = \tau_j - \tau_{j-1} < \infty$ $(j \in N)$ is the impulsive interval, then the impulsive fuzzy control system (3) is globally asymptotically stable at the origin.

According to Theorem 1 and Corollary 1, once we choose the constant impulsive interval σ and the common control matrix K for any instant τ_j, a simple design strategy for estimating the bound on K can be obtained as follows.

Algorithm.

- Find a positive definite symmetric matrix P and a corresponding scalar γ of Theorem 1 by choosing any positive scalar κ in advance and solving the linear matrix inequality (4) based on LMI technique.
- Choose a suitable impulsive interval σ according to the needs of the actual applications and a constant $\xi > 1$. In term of (6) or (12), compute the bound on β and estimate the bound on $I + K$ by using (5) or (11), i.e. by applying the inequality

$$\|I + K\| < \lambda_{\min}(P)^* \exp(-\gamma\sigma)/(\lambda_{\max}(P)\xi). \tag{14}$$

We could use (14) in order to find the control matrix K. For example, we may choose a diagonal matrix K such that (14) holds. Furthermore, notice that the designed procedure mentioned above could be reconstructed by basing it on Theorem 2 and Corollary 2 as we did before.

4. Experimental results

In this section, we illustrate the effectiveness of our results by showing several simulation results employing the Lorenz, Chua's and Rossler systems.

Example 1. Lorenz's system

$$\begin{cases} \dot{x}_1(t) = -ax_1(t) + ax_2(t), \\ \dot{x}_2(t) = cx_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t), \end{cases}$$

where a, b, and c are constants. Assume that $x_1(t) \in [-d, d]$ and $d > 0$. By (2) we have the following impulsive fuzzy control model.

Rule i: If $x_1(t)$ is M_i, then $\begin{cases} \dot{x}(t) = A_i x(t) & t \neq \tau_j, \\ \Delta x = K_{i,j}x(t) & t = \tau_j, \end{cases}$ $i = 1,2$ and $j \in N$, where $x(t) = [x_1(t), x_2(t), x_3(t)]^T$,

$$A_1 = \begin{bmatrix} -a & a & 0 \\ c & -1 & -d \\ 0 & d & -b \end{bmatrix}, \quad A_2 = \begin{bmatrix} -a & a & 0 \\ c & -1 & d \\ 0 & -d & -b \end{bmatrix}, \quad M_1(x_1(t)) = 0.5^*\left(1 + \frac{x_1(t)}{d}\right), \quad \text{and} \quad M_2(x_1(t)) = 0.5^*\left(1 - \frac{x_1(t)}{d}\right).$$

In this paper, we choose $a = 10$, $b = 8/3$, $c = 28$, and $d = 30$. According to Theorem 1 and Corollary 1, we set $\sigma = 0.05$, $\xi = 1.2$, $\kappa = 200$. Based on the algorithm of Theorem 1 or Corollary 1, we may deduce that $P = [14.9846, 2.5250, -0.0000; 2.5250, 3.2273, -0.0000; -0.0000, -0.0000, 5.1722]$, $\lambda_{\max}(P) = 15.5039$, $\lambda_{\min}(P) = 2.7080$, $\gamma = 73.8552$, and $\beta = 0.0057$. It follows that $\|I + K\| < 0.0036$. Thus we may choose $K_{i,j} = \mathrm{diag}([-0.999, -0.999, -0.999])$ $(j \in N)$ such that

$$\ln(\xi\beta) + \gamma\sigma = -1.2877 < 0.$$

Fig. 1a shows a solution trajectory of the Lorenz system in the state space before applying any control strategy, while Fig. 1b shows the control result of fuzzy Lorenz system starting at the initial condition $[-0.2, 0.6, 0.8]$. Clearly, the time series of all the variables of the Lorenz system converge to zero. Similar control results can be obtained for Chen's and Lü systems.

Example 2. Chua's oscillator

$$\begin{cases} \dot{x}_1(t) = 10(-x_1(t) + x_2(t) - f(x_1(t))), \\ \dot{x}_2(t) = x_1(t) - x_2(t) + x_3(t), \\ \dot{x}_3(t) = 14.87x_2(t), \end{cases} \qquad f(x_1(t)) = -0.68x_1(t) - 0.295(|x_1(t) + 1| - |x_1(t) - 1|).$$

By (2) we have the following impulsive fuzzy control model:

Rule i: If $x_1(t)$ is M_i, then $\begin{cases} \dot{x}(t) = A_i x(t) & t \neq \tau_j, \\ \Delta x = K_{i,j} x(t) & t = \tau_j, \end{cases}$ $i = 1, 2$ and $j \in N$, where $x(t) = [x_1(t), x_2(t), x_3(t)]^{\mathrm{T}}$,

$$A_1 = \begin{bmatrix} 10(d-1) & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -10(d+1) & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -14.87 & 0 \end{bmatrix},$$

$$\phi(x_1(t)) = \begin{cases} f(x_1(t))/x_1(t) & x_1(t) \neq 0, \\ -0.27 & x_1(t) = 0, \end{cases} \quad M_1(x_1(t)) = 0.5(1 - \phi(x_1(t))/d), \quad M_2(x_1(t)) = 1 - M_1(x_1(t)),$$

In this paper, we choose $d = 3$, $\sigma = 0.005$, $\xi = 1.2$, $\kappa = 200$, then $P = [2.6269, -1.6108, -1.0936; -1.6108, 12.4114, 5.8169; -1.0936, 5.8169, 183.7264]$, which implies that $\lambda_{\max}(P) = 183.9310$, $\lambda_{\min}(P) = 2.3684$, $\gamma = 84.4452$, $\beta = 0.0777$ $(j \in N)$. Thus, we have

$$|I + K| < 0.007.$$

From the inequality above we can choose $K_{i,j} = \mathrm{diag}([-0.999, -0.999, -0.999])$ $(j \in N)$, such that

$$\ln(\xi\beta) + \gamma\sigma = -1.9509 < 0.$$

Fig. 2a shows a solution trajectory of the Chua's oscillator in the state space before applying any control strategy, while Fig. 2b shows the control result of fuzzy Chua's oscillator starting at the initial condition $[-0.2, 0.6, 0.8]$. Clearly, the time series of all the variables of this system converge to zero.

Example 3. Rossler system

$$\begin{cases} \dot{x}_1(t) = -x_2(t) - x_3(t), \\ \dot{x}_2(t) = x_1(t) + ax_2(t), \\ \dot{x}_3(t) = bx_1(t) - \{c - x_2(t)\}x_3(t), \end{cases}$$

where a, b, and c are constants. Assume that $x_1(t) \in [c - d, c + d]$ and $d > 0$. Then we can have the following impulsive fuzzy control model.

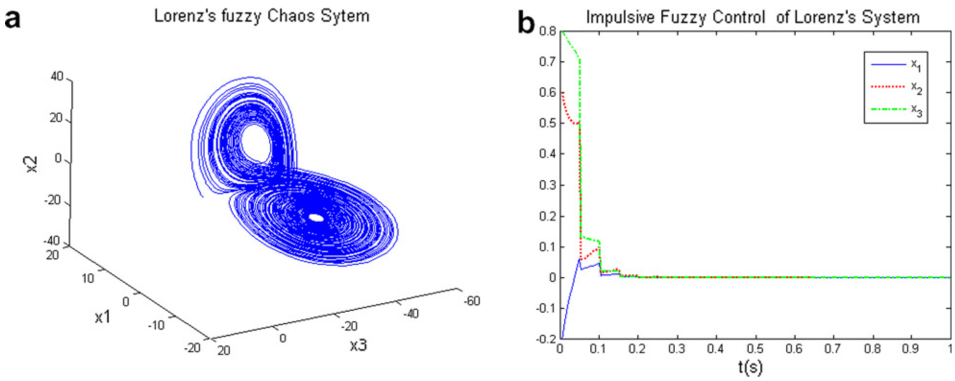

Fig. 1. (a) Solution trajectory of the Lorenz system and (b) impulsive control result of fuzzy Lorenz system.

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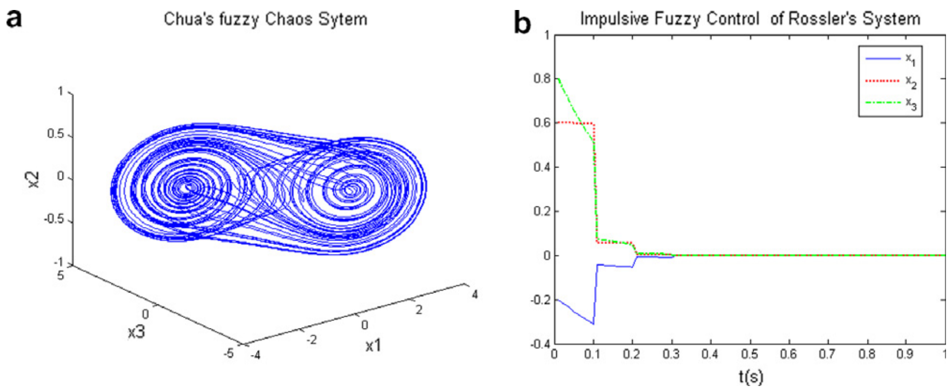

Fig. 2. (a) Solution trajectory of fuzzy Chua's oscillator and (b) impulsive control result of fuzzy Chua's oscillator.

Rule *i:* If $x_1(t)$ is M_i, then $\begin{cases} \dot{x}(t) = A_i x(t) & t \neq \tau_j, \\ \Delta x = K_{i,j} x(t) & t = \tau_j, \end{cases}$, $i = 1,2$ and $j \in N$, where $x(t) = [x_1(t), x_2(t), x_3(t)]^T$,

$$A_1 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & -d \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & -1 & -1 \\ 1 & a & 0 \\ b & 0 & d \end{bmatrix},$$

$$M_1(x_1(t)) = 0.5^* \left(1 + \frac{c - x_1(t)}{d}\right), \quad M_2(x_1(t)) = 0.5^* \left(1 - \frac{c - x_1(t)}{d}\right),$$

In this paper, $a = 0.34$, $b = 0.4$, $c = 4.5$ and $d = 10$. In order to impulsively control Rossler system we choose the remaining parameters as $\sigma = 0.005$, $\xi = 1.2$, $\kappa = 200$. Thus according to the proposed algorithm we have $P = [74.1513, -7.8313, -1.4215;\ -7.8313, 65.9990, 0.4228;\ -1.4215, 0.4228, 3.4211]$, which implies that $\lambda_{\max}(P) = 78.9310$, $\lambda_{\min}(P) = 3.3914$, $\gamma = 58.9727$, $\beta = 0.0233 (j \in N)$. Thus we have $\|I + K\| < 0.0267$. Based on the inequality above, $K_{i,j} = \mathrm{diag}([-0.999, -0.999, -0.999])$ $(j \in N)$ such that

$$\ln(\xi\beta) + \gamma\sigma = -3.2832 < 0.$$

Fig. 3a shows a solution trajectory of the Rossler system in the state space before applying any control strategy, while Fig. 3b shows the control result of fuzzy Rossler system starting at the initial condition $[-0.2, 0.6, 0.8]$. Clearly, the time series of all the variables of this system converge to zero.

In summary, according to the above simulation experiments, we think that the proposed control model for chaotic systems and the corresponding control methods can provide an effective method controlling chaos.

5. Conclusion

In this paper, an impulsive T-S fuzzy framework for controlling T-S fuzzy chaotic systems has been proposed through combining impulsive control technique with intelligent fuzzy logic methodology. The proposed control methods have the many advantages, such as, small impulse control, fuzzy intelligence and unified control way. Then some new and less conservative criteria have been derived based on Lyapunov method to guarantee the global asymptotic stability of the impulsive

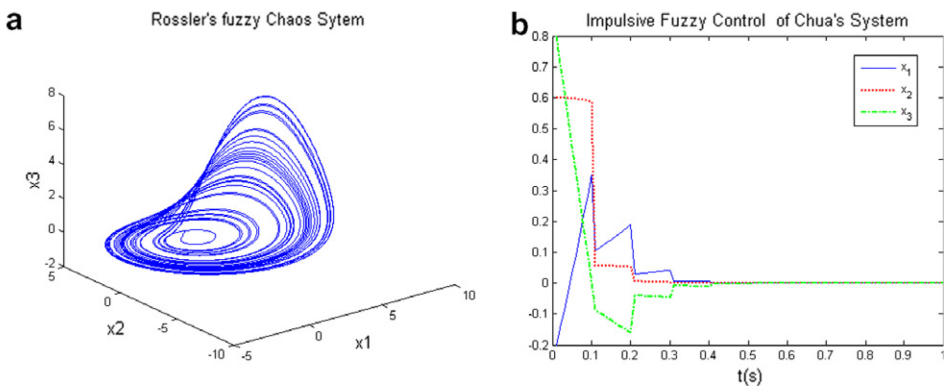

Fig. 3. (a) Solution trajectory of fuzzy Rossler system and (b) impulsive control result of fuzzy Rossler system.

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fuzzy model. The design procedures for estimating the bound of control matrices proposed in this paper have the virtues of simplicity and computational attractiveness. The effectiveness of our methods has been demonstrated through numerical simulations of a class of chaotic systems including Lorenz, Chua's and Rossler systems.

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