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Analysis and design for unified exponential stability of three different impulsive T–S fuzzy systems

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ABSTRACT

Three different forms of impulsive T–S models are discussed in this paper; the first one is described by a nonlinear impulsive control system represented by T–S model, while the second one is expressed as a state feedback impulsive control plant and the third one is depicted by a hybrid system. A simple and unified Lyapunov-based stability criterion is proposed to guarantee the exponential stability of closed-loop impulsive fuzzy systems. Such criterion is expressed in the form of linear matrix inequalities and the corresponding design algorithms are presented. Several numerical simulations are shown to demonstrate how the proposed controllers can stabilize these impulsive fuzzy systems.

1. Introduction

It has been shown recently that fuzzy-logic is a successful control methodology, quite effective in controlling plants that are complex, uncertain and ill-defined, with well-developed qualitative design methods available in the literature. Since the work of Tanaka and Sugeno [1] on the stability of state feedback controllers, there has been an extensive research effort in developing system theory for various types of T–S models (see [1–11 and the references therein). Impulsive control, on the other hand, has been a very promising technique for stabilizing chaotic systems using only small control impulses. It offers a direct method to modulating digital information onto a chaotic carrier signal for spread spectrum applications (see [12–15 and the references therein). More recently, a novel model based on T–S models and impulsive systems has been developed. It has been found out that the local dynamics of this type of impulsive T–S models at different regions of the state space can be described by linear impulsive systems. The basic idea of such models is that impulsive plant is introduced into a nonlinear system represented by fuzzy T–S model and the overall behavior of the system is regulated by fuzzy blending of these linear impulsive models. For example, in [16,17], the unified control approaches that integrated intelligent fuzzy logic methodology and impulsive control have been presented for controlling a class of chaotic systems. In [20], methods based on the classical Razumikhin technique have been applied to obtain criteria for uniform stability and uniform asymptotic stability of T–S fuzzy delay systems with impulses. An interesting application of this theory to the impulsive synchronization of T–S fuzzy models by using continuous chaotic systems has been conducted in [18], while in [19], a criterion for the exponential stability of chaotic systems impulsively controlled by fuzzy T–S models have been also derived. In [21], authors have proposed an impulsive control scheme for discrete T–S fuzzy systems. The main advantage of these impulsive fuzzy models is that it can tackle various stability properties of nonlinear systems by combining impulsive control techniques with fuzzy logic methodology.

It is important to point out that the models discussed in [16–19,21] are restricted to chaos control and lack generality. The lack of generality is due to the fact that an input-free variable has to be imposed on the special plant discussed in [16–21].

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Moreover, if there exist some external input terms in these practical systems or if the plant itself is impulsive, the results of [16–21] become inapplicable. The conditions obtained are concerned with asymptotic stability, more conservative and do not translate into LMIs (except for one result in [16,19] which has an LMI form and applies only to special cases). It is quite essential to analyze exponential stability of these models due to its fast convergence and desirable accuracy when compared to asymptotic stability.

Motivated by the results in [16–21], we develop three different forms for impulsive fuzzy model and construct here a unified control design methods for the proposed model, where both fuzzy dynamic models and impulsive techniques are used. The main contributions of this paper can be summarized as follows: (1) First, we extend the impulsive fuzzy T–S model in [16–21] to a more generalized form in which wither (a) the control input are impulses, (b) the system plant itself is impulsive, or (c) the whole system is expressed as a hybrid system (Section 2). (2) Second, we derive sufficient conditions for unified exponential stability of three different forms of the impulsive fuzzy model by using PDC techniques, some of which are cast as LMIs (Section 3.1). (3) Finally, we design several iterative algorithms aimed at stabilizing these impulsive fuzzy systems based on the proposed criteria for exponential stability (Section 3.2). These algorithms are based on a set of LMIs that are easily solvable numerically by using commercially available software.

2. Preliminaries

Consider the following nonlinear system with both impulsive effects and external inputs represented by T–S fuzzy model.

Plant Rule i: IF $z_1(t)$ is $M_{i1}, \ldots, z_p(t)$ is M_{ip}.

$$\text{THEN}\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) & t \neq \tau_k, \\ \Delta x|_{t=\tau_k} = x(t_k^+) - x(t_k^-) \equiv U_i(k, x) & t = \tau_k, \quad i = 1, 2, \ldots, r, \quad k = 1, 2, \ldots, \\ x(t_0^+) = x_0, \end{cases} \tag{1}$$

where $z_1(t) \sim z_p(t)$ are the premise variables, M_{in} is the fuzzy set and r is the number of IF–THEN rules, $x(t) \in R^m$ is the state, $u(t) \in R^m$ denotes the input variable, $A_i \in R^{n\times m}$, $B_i \in R^{n\times m}$, and the discrete set of impulsive moments $\{\tau_j\}$ satisfies

$$0 \leqslant \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_k < \tau_{k+1} < \cdots, \quad \tau_k \to \infty \text{ as } k \to \infty.$$

Remark 1. According to (1), we have three possible cases to consider: Either (a) $u(t) = 0$ and the impulses are controller to be designed, (b) impulses are viewed as impulsive perturbations and $u(t)$ is an external control input, or (c) the impulses and $u(t)$ are viewed as hybrid controllers to be designed. In the following, we present a unified theory to deal with the three cases.

By using a singleton fuzzifier, product inference and a center-average defuzzifier, the following dynamic model can be obtained from (1).

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\}, & t \neq \tau_k, \\ \Delta x = \sum_{i=1}^{r} h_i(z(t)) U_i(t_k, x), & t = \tau_k, \\ x(t_0^+) = x_0, \end{cases} \tag{2}$$

where $z(t) = [z_1(t), \; z_2(t), \ldots, z_p(t)]$ is the premise vector, $h_i(z(t)) = w_i(z(t))/\sum_{i=1}^{r} w_i(z(t))$ for all t, $w_i(z(t)) = \prod_{n=1}^{p} M_{in}(z_k(t)), \sum_{i=1}^{r} w_i(z(t)) > 0, w_i(z(t)) \geqslant 0, i = 1, 2, \ldots, r, M_{ik}$ is the fuzzy set ($i = 1,2, \ldots ,r, k = 1,2, \ldots ,p$). From these formulas we have $\sum_{i=1}^{r} h_i(z(t)) = 1, h_i(z(t)) \geqslant 0, i = 1, 2, \ldots, r$.

In terms of the conventional PDC technique, the input variables satisfy the following rules.

Rule i: IF $z_1(t)$ is $M_{i1}, \ldots, z_p(t)$ is M_{ip},

$$\text{THEN } u(t) = -K_i x(t). \tag{3}$$

According to the defuzzifying method applied on (1), state feedback control law (3) becomes

$$u(t) = -\frac{\sum_{i=1}^{r} w_i(z(t)) K_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = -\sum_{i=1}^{r} h_i(z(t)) K_i x(t). \tag{4}$$

Substituting (4) into (2) yields a closed-loop system described by the state space model

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t))(A_i - B_i K_j) x(t), & t \neq \tau_k, \\ \Delta x = \sum_{i=1}^{r} h_i(z(t)) U_i(t_k, x), & t = \tau_k. \end{cases} \tag{5}$$

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Let $f(t,x) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i - B_iK_j)x(t)$ and $I_k(t_k,x) = \sum_{i=1}^{r} h_i(z(t))U_i(t_k,x)$, thus $f(t,0) = 0$, $I_k(t,0) = 0$. Moreover, by letting $f(t,x)$ satisfy Lipschitz condition, then system (5) admits the trivial solution.

Remark 2. In (5), only if $K_j = 0$, we may think of (5) as the first case. Hence, we think of (5) extending the existing impulsive fuzzy systems.

Definition 1. Let $V{:}R_+ \times R^n \to R_+$, then V is said to belong to class v_0, if

(i) V is continuous in $(\tau_{j-1},\tau_j] \times R^n$ and for each $x \in R^n,\ j = 1,2,\ldots,\lim_{(t,y)\to(\tau_j^+,x)} V(t,y) = V(\tau_j^+,x)$ exists.
(ii) V is locally Lipschitz in x.

Definition 2. For $(t,x) \in (\tau_{j-1},\tau_j] \times R^n$, we define

$$D^+V(t,x) \equiv \lim_{h\to 0^+} \sup \frac{1}{h}\{V[t+h, x+hf(t,x)] - V(t,x)\}.$$

Definition 3. $\|\cdot\|$denotes the Euclidean norm on R^n.

Definition 4. The trivial solution of system (5) is said to be exponentially stable, if for any initial data $x(t_0)$, there exists an $\alpha > 0$, and for every $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that $\|x(t,t_0,x(t_0)\| < \exp(-\alpha(t-t_0))$, for all $t \geqslant t_0 \geqslant 0$.

3. Main results

In this section, we shall initially state and prove the general results of system (5), then give suitable design methods to stabilize (5).

3.1. Stability criterion

Theorem 3.1. *Assume that there exist $V \in v_0$, constants p, α, l, c, c_1, $c_2 > 0$, $d_k \geqslant 0$, $k \in N$, such that*

(i) $c_1\|x\|^P \leqslant V(t,x) \leqslant c_2\|x\|^P$, $t \in R_+$, $x \in R^n$;
(ii) for each $k \in N$ and $x \in R^n, V(t_k, x + I_k(t_k,x)) \leqslant d_k V(t_k^-, x)$;
(iii) for all $t \in [\tau_{k-1},\tau_k), k \in N, \dot{V}(t,x) \leqslant cV(t,x)$;
(iv) for any $k \in N$, $0 < \tau_{k+1} - \tau_k \leqslant m$, $\ln d_k \leqslant -(\alpha + c)m$.

Then the trivial solution of (5) is exponentially stable.

Proof. Let $x(t) \triangleq x(t,t_0,x_0)$ be any solution of (5) whose initial condition satisfies the inequality $\|x(t_0)\| < \delta$. For any given $\varepsilon \in (0,1]$, choose $\delta = \delta(\varepsilon) > 0$, such that $c_2\delta^q < c_1\varepsilon^q \exp(-(\alpha + c)m)$.

$$\text{From condition (iii), we have } v(t) \leqslant v(\tau_{k-1})\exp(-c(t-\tau_{k-1})), \quad \text{for } t \in [\tau_{k-1},\tau_k),\ k \in N. \tag{6}$$

We shall prove that $v(t) < c_1\varepsilon^q \exp(-(\alpha + c)km)\exp(c(t - t_0))$, and $\|x(t)\| < \varepsilon\exp\left(-\frac{\alpha}{q}(t-t_0)\right)$, for any $t \in [\tau_{k-1},\tau_k)$, $k \in N$.

For $k = 1$, we have, according to conditions (i) and (iv),

$$v(t) \leqslant v(t_0)\exp(c(t-t_0)) \leqslant c_1\varepsilon^q\exp(-(\alpha+c)m)\exp(c(t-t_0)),$$

thus

$$\|x(t)\|^q \leqslant \frac{1}{c_1}v(t) \leqslant \varepsilon^q\exp(-(\alpha+c)m)\exp(c(\tau_1-t_0)) \leqslant \varepsilon^q\exp(-\alpha(t-t_0)), \quad t \in [\tau_0,\tau_1).$$

Hence

$$\|x(t)\| \leqslant \varepsilon\exp(-\alpha(t-t_0)/q), \quad t \in [\tau_0,\tau_1). \tag{7}$$

Let's now apply mathematical induction. Suppose that (6) holds for $k = j$, i.e.,

$$\begin{aligned} &v(t) < c_1\varepsilon^q\exp(-(\alpha+c)jm)\exp(c(t-t_0)),\ \text{and} \\ &\|x(t)\| < \varepsilon\exp\left(-\frac{\alpha}{q}(t-t_0)\right), \quad \text{for } t \in [\tau_{j-1},\tau_j),\ j \geqslant 2. \end{aligned} \tag{8}$$

Now let's prove that (8) holds for $k = j + 1$. From condition (i) and (8), we have for $t \in [\tau_{j-1},\tau_j)$

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$$\|x(t)\|^q \leqslant \frac{1}{c_1} v(t) \leqslant \varepsilon^q \exp(-(\alpha+c)jm)\exp(c(\tau_j - t_0)).$$

Thus $\|x(t)\| \leqslant \varepsilon \exp(-(\alpha+c)jm/q)\exp(c(\tau_j - t_0)/q)$. By condition (ii), we obtain $v(\tau_j) \leqslant d_j c_1 \varepsilon^q \exp(-(\alpha+c)jm)\exp(c(\tau_j - t_0))$. It follows that according to condition (ii), we have

$$\begin{aligned} v(\tau_j) &\leqslant d_j c_1 \varepsilon^q \exp(-(\alpha+c)jm)\exp(c(\tau_j - t_0)) \\ &\leqslant c_1 \varepsilon^q \exp(-(\alpha+c)(j+1)m)\exp(c(\tau_j - t_0)). \end{aligned} \tag{9}$$

By (6) and (9) and for $t \in [\tau_j, \tau_{j+1})$, we get $v(t) \leqslant v(\tau_j)\exp(c(t-\tau_j)) < c_1\varepsilon^q \exp(-(\alpha+c)(j+1)m)\exp(c(t-t_0))$. Thus

$$\begin{aligned} \|x(t)\|^q &\leqslant \varepsilon^q \exp(-(\alpha+c)(j+1)m)\exp(c(t-t_0)) \leqslant \varepsilon^q \exp(-(\alpha+c)(j+1)m)\exp(c(\tau_{j+1}-t_0)) \\ &\leqslant \varepsilon^q \exp(-(\alpha+c)(j+1)m)\exp(c(j+1)m) \leqslant \varepsilon^q \exp(-(j+1)\alpha m) \\ &\leqslant \varepsilon^q \exp(-(\tau_{j+1}-\tau_j+\tau_j-\tau_{j-1}+\cdots+\tau_1-t_0)\alpha) \leqslant \varepsilon^q \exp(-(\tau_{j+1}-t_0)\alpha) \\ &\leqslant \varepsilon^q \exp(-(t-t_0)\alpha), \quad t \in [\tau_j, \tau_{j+1}) \end{aligned}$$

which implies that (8) holds for $k = j + 1$. In other words, (8) holds for all $k \in N$. Hence $\|x(t)\| \leqslant \varepsilon e^{-\alpha(t-t_0)/q}$, for $t \geqslant t_0$, i.e., the trivial solution of system (5) is exponentially stable. □

Remark 3. The results obtained here are applicable to the three cases discussed earlier. In fact, condition (iii) allows $\dot{V}(t) > 0$, for all $t \in R^+$, which means that the nonlinear fuzzy T–S model, may be unstable. In other words, the impulses made the underlying system become stable.

In order to obtain the stability design algorithm for (5), we shall assume, without loss of generality, that $U_i(t_k,x) = D_{ik}x$. It should be noted here that $\sum_{i=1}^r h_i(z(t))D_{i,k}x(t)$ is the complex nonlinear term that may have impulsive structure. Therefore, let's rewrite (5) as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^r h_i^2(z(t))G_{ii}x(t) + 2\sum_{i<j}^r h_i(z(t))h_j(z(t))\left(\frac{G_{ij}+G_{ji}}{2}\right)x(t), & t \neq \tau_k, \\ \Delta x = \sum_{i=1}^r h_i(z(t))D_{i,k}x(t), & t = \tau_k, \end{cases} \tag{10}$$

where $G_{ij} = A_i - B_iK_j$ ($i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, r$).

Theorem 3.2. *If there exist a positive definite symmetric matrix P and constants c, $\alpha > 0$, such that*

(i) $G_{ii}^T P + PG_{ii} - \frac{1}{2}cP < 0, \quad i = 1,2,\ldots,r, \; j = 1,2,\ldots,r;$
(ii) $(G_{ij}+G_{ji})^T P + P(G_{ij}+G_{ji}) - \frac{1}{2}cP < 0, \quad i = 1,2,\ldots,r, \; j = 1,2,\ldots,r;$
(iii) *for any* $k \in N$, $0 < \tau_{k+1} - \tau_k \leqslant m$, $\ln(d_k) \leqslant -(\alpha+c)m$, *where* $d_k = \frac{\lambda_{\max}(P)\max_{1\leqslant i\leqslant r}\{\|(I+D_{ik})\|\}}{\lambda_{\min}(P)}$,

then the trivial solution of system (10) is exponentially stable.

Proof. Choose the Lyapunov function $V(t,x) = x(t)^T Px(t)$, where $P > 0$, for $t \neq \tau_k$. In this case,

$$D^+V(t,x) = \sum_{i=1}^r h_i^2(z(t))x(t)^T(G_{ii}^T P + PG_{ii})x(t) + 2\sum_{i<j}^r h_i(z(t))h_j(z(t))x(t)^T\left[\left(\frac{G_{ij}+G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij}+G_{ji}}{2}\right)\right]x(t). \tag{11}$$

By applying conditions (i) and (ii), and noting that $0 \leqslant h_i(z(t)) \leqslant 1$ and $\sum_{i=1}^r h_i(z(t)) = 1$, we get $D^+V(t,x) \leqslant cV(t,x)$.
For $t = \tau_k$,

$$\begin{aligned} &V\left(t_k^+, x(t_k) + \sum_{i=1}^r h_i(z(t))D_{ik}x(t_k)\right) \\ &= \left[x(t_k) + \sum_{i=1}^r h_i(z(t))D_{ik}x(t_k)\right]^T P\left[x(t_k) + \sum_{i=1}^r h_i(z(t))D_{ik}x(t_k)\right] \\ &= x(t_k)^T\left(I + \sum_{i=1}^r h_i(z(t))D_{ik}\right)^T P\left(I + \sum_{i=1}^r h_i(z(t))D_{ik}\right)x(t_k) \leqslant \lambda_{\max}(P)\|I + \sum_{i=1}^r h_i(z(t))D_{ik}\|\|x(t_k)\| \\ &= \lambda_{\max}(P)\left(\sum_{i=1}^r h_i(z(t))\|(I+D_{ik})\|\right)\|x(t_k)\| \leqslant \frac{\lambda_{\max}(P)\max_{1\leqslant i\leqslant r}\{\|(I+D_{ik})\|\}}{\lambda_{\min}(P)}V(t_k^+, x(t_k)) = d_kV(t_k^+, x(t_k)). \end{aligned} \tag{12}$$

According to Theorem 3.1, (11) and (12), it follows that the trivial solution of system (10) is exponentially stable.

Remark 4. According to condition (iii), an upper bound on the time interval between two consecutive impulses is the only required quantity to be known.

When $D_{i,k} = D$, then the impulsive term in (10) becomes linear. In this case, we have the following simple result.

Corollary 3.1. *If there exist a positive definite symmetric matrix P and constants c,α > 0, such that*

(i) $G_{ii}^T P + PG_{ii} - \frac{1}{2}cP < 0, \quad i = 1,2,\dots,r, \; j = 1,2,\dots,r;$
(ii) $(G_{ij} + G_{ji})^T P + P(G_{ij} + G_{ji}) - \frac{1}{2}cP < 0, \quad i = 1,2,\dots,r, \; j = 1,2,\dots,r;$
(iii) for any $k \in N$, $0 < \tau_{k+1} - \tau_k \leqslant m$, $\ln d \leqslant -(\alpha + c)m$, *where* $d = \frac{\lambda_{\max}(P)\|I+D\|}{\lambda_{\min}(P)}$,

then, the trivial solution of system (10) is exponentially stable.

It should be mentioned here that conditions (i) and (ii) of Theorem 3.2 and Corollary 3.1 are not convex optimization conditions, therefore we may define $X = P^{-1}$, $M_i = K_i X$, and multiply conditions (i) and (ii) by X, to obtain

$$XA_i^T + A_i X - M_i^T B_i^T - B_i M_i - cX/2 < 0, \tag{13}$$
$$XA_i^T + A_i X + XA_j^T + A_j X - M_j^T B_i^T - B_i M_j - M_i^T B_j^T - B_j M_i - cX/2 < 0. \tag{14}$$

Obviously, (13) and (14) are linear matrix inequalities with respect to X and M_i. Thus we may transform inequalities (13) and (14) into the following general eigenvalue problem.

$$\underset{X,M_1\dots M_r}{\text{Maximize}}\, c$$
$$\begin{cases} \text{subject to } X > 0 \\ XA_i^T + A_i X - M_i^T B_i^T - B_i M_i - cX/2 < 0 \\ \text{for all } i \text{ and } i < j \\ XA_i^T + A_i X + XA_j^T + A_j X - M_j^T B_i^T - B_i M_j - M_i^T B_j^T - B_j M_i - cX/2 < 0. \end{cases} \tag{15}$$

When (1) is external-input-free, i.e., (1) is a fuzzy system with impulsive control input, we have the following expression.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))A_i x(t), \quad t \neq \tau_k, \\ \Delta x = \sum_{i=1}^{r} h_i(z(t))D_{i,k} x(t), \quad t = \tau_k. \end{cases} \tag{16}$$

Corollary 3.2. *If there exist a positive definite symmetric matrix P and constants c, α > 0, such that*

(i) $A_i^T P + PA_i - \frac{1}{2}cP < 0, \quad i = 1,2,\dots,r, \; j = 1,2,\dots,r;$
(ii) for any $k \in N$, $\ln(d_k) \leqslant -(\alpha + c)m$, *where* $d_k = \frac{\lambda_{\max}(P)\max_{1\leqslant i\leqslant r}\{\|(I+D_{ik})\|\}}{\lambda_{\min}(P)}$,

then, the trivial solution of system (16) is exponentially stable.

3.2. Algorithm design

In the following procedure, we shall distinguish between the three previously discussed cases in order to develop a stabilizing design algorithm for (1).

Case 1. System (1) has no external continuous input and the impulses are regarded as control input. In this simple case, we choose the control gain matrix $D_{i,k}$ to satisfy Corollary 3.2, in such a way that $\max_{1\leqslant i\leqslant r}\{\|(I + D_{ik})\|\} \leqslant \lambda_{\min}(P)e^{-(\alpha+c)l}/\lambda_{\max}(P)$, where P is the solution to condition (i) of Corollary 3.2.

Case 2. System (1) contains both the impulses and external continuous control inputs. In other words, system (1) is a hybrid system containing both continuous and impulsive control inputs. According to Theorem 3.2 or Corollary 3.1, we have the following design procedure.

Step 1. Initialize constant α and c.
Step 2. By (13) and (14), figure out common P and state feedback matrix K_i.
Step 3. By condition (iii) of Theorem 3.2, an estimate on $D_{i,k}$ is obtained by applying

$$\max_{1\leqslant i\leqslant r}\{\|(I + D_{ik})\|\} \leqslant \lambda_{\min}(P)e^{-(\alpha+c)l}/\lambda_{\max}(P).$$

Case 3. System (1) is itself impulsive, with external continuous control input. In this case, the state feedback matrix K is the control matrix to be designed, while $D_{i,k}$ and l are given in advance. Without loss of generality, we let P to be a scalar matrix, i.e., $\lambda_{\min}(P) = \lambda_{\max}(P)$. Then, we have

Step 1. By condition (iii) of Theorem 3.2, provide an estimate for $(\alpha + c)$, by using the inequality $0 < (\alpha + c) < -\ln(d_k)/m$.
Step 2. By Step 1, choose two positive constant α and c.
Step 3. By (13) and (14), figure out K_i.

4. Simulation experiments and discussions

We shall consider here the Lorenz system with impulses and external input terms, given by

$$\begin{cases} \dot{x}_1(t) = -10x_1(t) + 10x_2(t) + u_1(t), \\ \dot{x}_2(t) = 28x_1(t) - x_2(t) - x_1(t)x_3(t) + u_1(t), \quad t \neq \tau_k, \\ \dot{x}_3(t) = x_1(t)x_2(t) - \frac{8}{3}x_3(t) + u_1(t), \\ \Delta x_1(t) = x_1(t) - d_{11}x_1(t), \\ \Delta x_2(t) = x_2(t) - d_{22}x_2(t), \\ \Delta x_3(t) = x_3(t) - d_{33}x_3(t), \quad t = \tau_k, \end{cases} \tag{17}$$

where d_{ii} $(i = 1,2,3)$ are the quantities to be estimated. Assume that $x_1(t) \in [-d, d]$ and $d > 0$. Then the impulsive fuzzy control model is given by

Plant Rule i: IF $x_1(t)$ is M_i.

$$THEN \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), & t \neq \tau_k, \\ \Delta x|_{t=\tau_k} = x(t_k^+) - x(t_k^-) \equiv D_i x(t), & t = \tau_k, \quad i = 1,2, \; k = 1,2,\ldots \\ x(t_0^+) = x_0, \end{cases} \tag{18}$$

where

$$A_1 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & -d \\ 0 & d & 8/3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -10 & 10 & 0 \\ 28 & -1 & d \\ 0 & -d & 8/3 \end{bmatrix}, \quad D_i = \begin{bmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{bmatrix},$$

$i = 1,2.$ $\quad M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{x_1(t)}{d}\right), M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{x_2(t)}{d}\right)$, and let $d = 30$.

We shall discuss the stability of system (18) according to the three cases mentioned above.

Case 1. Let $B_i = \mathbf{0}$. In this case, the impulses represent the control input of (18), and $d_{ii}(i = 1,2,3)$ are the quantities to be estimated. Let $m < 0.02$, $c = 25$ and $\alpha = 10$. According to Corollary 3.1, we have $P = \begin{bmatrix} 0.0146 & 0.00730.0000 \\ 0.0073 & 0.0243 - 0.000 \\ 0.0000 & -0.00000.0206 \end{bmatrix}$, $\lambda_{\max} = 0.0282$, $\lambda_{\min} = 0.0107$ and $\max_{1 \leqslant i \leqslant r}\{\|1 + d_{ii}\|\} \leqslant \lambda_{\min}(P)e^{-(\alpha+c)m}/\lambda_{\max}(P) = 0.1881$. Thus we may choose $d_{11} = d_{22} = d_{33} = -0.82$. So condition (ii) of Corollary 3.2 holds.

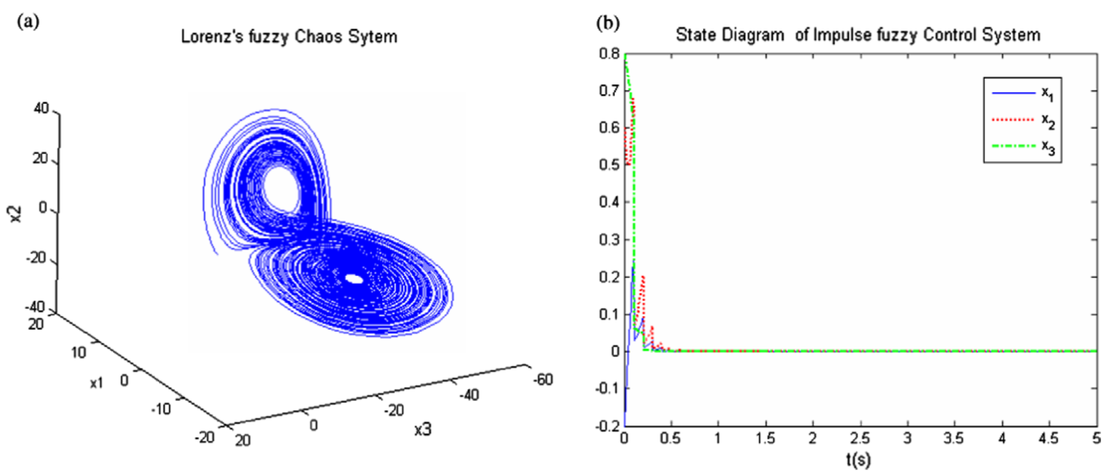

Fig. 1. Phase and state diagrams of impulsive fuzzy system. (a) Phase diagram of system without any control. (b) State diagram of system under impulsive control.

Fig. 1(a) and (b) show the phase diagrams of (18) without any control input and the time series of the solution under impulsive control, respectively. We could see from clearly from (b) that the solution trajectories of (18) exponentially converge to the equilibrium.

Case 2. We regard (18) as a hybrid system with two control inputs, one is impulsive, while the other is continuous. Let B_i be a unit matrix and let $c = 0.5$, $\alpha = 20$ and $m < 0.02$. According to (13) and (14), we have

$$P = \begin{bmatrix} 0.0083 & 0.0000 & -0.000 \\ 0.0000 & 0.0083 & 0.000 \\ -0.0000 & 0.0000 & 0.0083 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -9.8571 & 1.4376 & 19.1755 \\ 36.5624 & -0.8571 & -47.9458 \\ -19.1755 & 47.9458 & -2.5238 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -9.8571 & 20.1041 & -2.0792 \\ 17.8959 & -0.8571 & -0.0041 \\ 2.0792 & 0.0041 & -2.5238 \end{bmatrix}$$

$\max_{1 \leqslant i \leqslant r}\{\|1 + d_{ii}\|\} \leqslant \lambda_{\min}(P)e^{-(\alpha+c)m}/\lambda_{\max}(P) = 0.8106$. Thus we may choose $d_{11} = d_{22} = d_{33} = -0.2$. It follows that, in this case, control conditions become less conservative.

Fig. 2(a) shows the time series of system (18) under a state feedback control input and without impulsive control. Although the conditions of Corollary 3.1 hold, Fig. 2(a) clearly demonstrate that system (1) is unstable. On the other hand, Fig. 2(b) shows that the solutions trajectories are exponentially stable in the hybrid case.

Case 3. The plant itself is impulsive with strictly continuous control input. In this case, we need to design a state feedback control. It is important to point out that m, d_{ii} and l of (18) are constants. Let B_i be a unit matrix, and $m > 0.2$, $d_{11} = d_{22} = d_{33} = -0.9$. Although the impulsive intervals are not large (0.2 and 1, respectively), Fig. 3 (a) and (b) show that the solutions trajectories are unstable due to the absence of state feedback control. For $d = \|I + D\| = 0.1$, we choose $m = 1$. Since $0 < \alpha + c < -\ln(0.1)/m = 2.3026$, we let $c = 0.000005$, which implies that $0 < \alpha < \ln d/m - c = 2.3026$. In other words, there exists an α such that condition (iii) of Theorem 3.1 or Corollary 3.1 holds. In term of (13) and (14), we have

$$P = \begin{bmatrix} 0.0212 & -0.0000 & -0.0000 \\ -0.0000 & 0.0212 & 0.0000 \\ -0.0000 & 0.0000 & 0.0212 \end{bmatrix}, \quad K_1 = \begin{bmatrix} -9.6667 & -8.1350 & -0.1237 \\ 46.1350 & -0.6667 & 0.6432 \\ 0.1237 & -0.6432 & -2.3333 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -9.6667 & 23.1203 & 4.7925 \\ 14.8797 & -0.6667 & 0.0024 \\ -4.7925 & -0.0024 & -2.3333 \end{bmatrix}.$$

Fig. 3(c) shows the solutions trajectories of (18) under state feedback control input converging exponentially to zero.

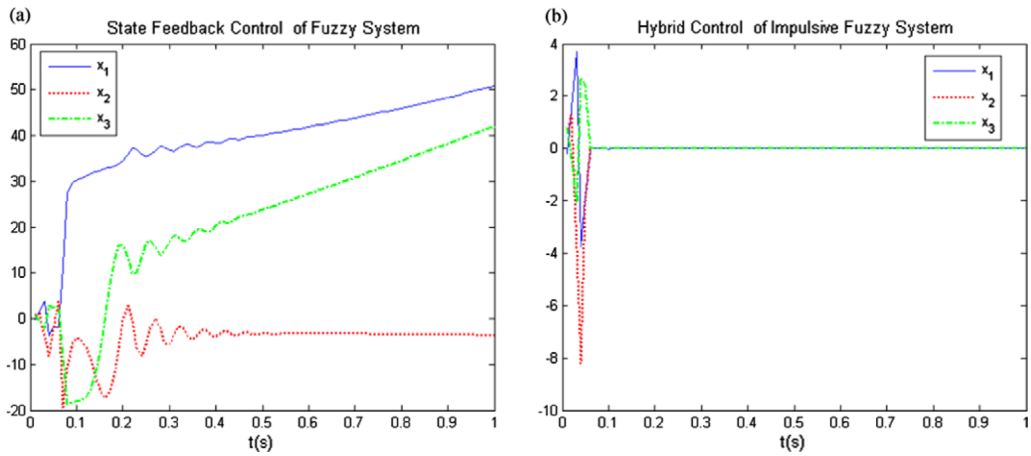

Fig. 2. State diagram of impulsive fuzzy system. (a) State diagram of system under state feedback control and without impulsive control. (b) State diagram of system under hybrid control of state feedback and impulse.

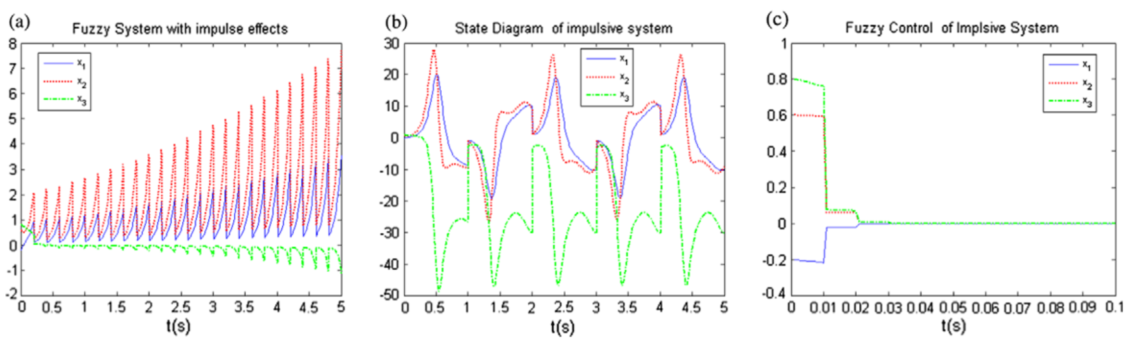

Fig. 3. State diagram of impulsive fuzzy system, (a) and (b) State diagram of impulsive system without state feedback control, (c) State diagram of impulsive system with state feedback control.

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5. Conclusions

We have extended ordinary T–S models to impulsive T–S models possessing external input terms. Such inputs can depict complex nonlinear systems under impulsive and/or state feedback control. We have obtained several new and unified sufficient conditions for the exponential stability of the proposed model under three different control inputs. The results obtained were applied to design an impulsive control or state feedback control procedure to achieve exponential stability.

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