

# Automatic detection of atrial fibrillation using the coefficient of variation and density histograms of RR and $\Delta$ RR intervals

K. Tateno L. Glass

Department of Physiology, McGill University, Montreal, Quebec, Canada

**Abstract**—The paper describes a method for the automatic detection of atrial fibrillation, an abnormal heart rhythm, based on the sequence of intervals between heartbeats. The RR interval is the interbeat interval, and  $\Delta$ RR is the difference between two successive RR intervals. Standard density histograms of the RR and  $\Delta$ RR intervals were prepared as templates for atrial fibrillation detection. As the coefficients of variation of the RR and  $\Delta$ RR intervals were approximately constant during atrial fibrillation, the coefficients of variation in the test data could be compared with the standard coefficients of variation (CV test). Further, the similarities between the density histograms of the test data and the standard density histograms were estimated using the Kolmogorov–Smirnov test. The CV test based on the RR intervals showed a sensitivity of 86.6% and a specificity of 84.3%. The CV test based on the  $\Delta$ RR intervals showed that the sensitivity and the specificity are both approximately 84%. The Kolmogorov–Smirnov test based on the RR intervals did not improve on the result of the CV test. In contrast, the Kolmogorov–Smirnov test based on the  $\Delta$ RR intervals showed a sensitivity of 94.4% and a specificity of 97.2%.

**Keywords**—Atrial fibrillation, RR interval, Coefficient of variation, Kolmogorov–Smirnov test

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## 1 Introduction

ATRIAL FIBRILLATION is a serious and common arrhythmia. Atrial fibrillation is associated with rapid, irregular atrial activation. The atrial activations are irregularly transmitted through the atrioventricular node, leading to a correspondingly irregular sequence of ventricular activations, as monitored by the ventricular interbeat (RR) intervals on the surface electrocardiogram (ECG). Clinically, in the surface ECG, atrial fibrillation is diagnosed by the absence of P-waves (normally associated with the near synchronous activation of the atria) and a rapid irregular ventricular rate. However, as P-waves are difficult to determine automatically, and irregular baseline activity of the ECG is common in atrial fibrillation, it is difficult automatically to detect atrial fibrillation from the surface ECG. This work presents an automatic method for atrial fibrillation detection based on the RR intervals.

As RR intervals during atrial fibrillation have a larger standard deviation and a shorter correlation length than those during normal sinus rhythm, the standard deviation and the autocorrelation can be used to distinguish atrial fibrillation from sinus

rhythm (BOOTSMA *et al.*, 1970). However, we also need to distinguish atrial fibrillation from other arrhythmias. As other arrhythmias often show irregular RR intervals, it is difficult to detect atrial fibrillation based solely on the RR intervals (ANDRESEN and BRÜGGEMANN, 1998; MURGATROYD *et al.*, 1995; PINCIROLI and CASTELLI, 1986; SLOCUM *et al.*, 1987).

Here,  $\Delta$ RR is defined as being the difference between successive RR intervals. The density histograms of the RR and  $\Delta$ RR intervals collected during atrial fibrillation are prepared as standard density histograms. The coefficients of variation of the RR and  $\Delta$ RR intervals computed from the standard density histograms are used to detect atrial fibrillation (CV test). Further, density histograms of RR or  $\Delta$ RR intervals in test data are compared with standard density histograms using the Kolmogorov–Smirnov test. If there is no significant difference between two given histograms, the rhythm is labelled as atrial fibrillation.

In the present work, we compare the CV test and the Kolmogorov–Smirnov test. Moreover, parameters of the Kolmogorov–Smirnov test need to be optimised. A preliminary report of our method appeared in TATENO and GLASS (2000).

## 2 Methods

Data were obtained from the MIT-BIH atrial fibrillation database (<http://www.physionet.org/physiobank/database/afdb>) and the MIT-BIH arrhythmia database (<http://www>.

Correspondence should be addressed to Dr L. Glass;  
email: [glass@cnd.mcgill.ca](mailto:glass@cnd.mcgill.ca)

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physionet.org/physiobank/database/mitdb). The atrial fibrillation database contains 300 atrial fibrillation episodes, sampled at 250 Hz for 10 h from Holter tapes of 25 subjects. The onset/end of atrial fibrillation was annotated by trained observers. The timing of each QRS complex was determined by an automatic detector.

The contents of the MIT-BIH atrial fibrillation database are summarised in Table 1. The MIT-BIH arrhythmia database includes two categories (the 100 series and the 200 series) and contains 48 subjects: The 100 series consists of 23 subjects, and the 200 series consists of 25 subjects. The 100 series includes normal sinus rhythm, paced rhythm, bigeminy, trigeminy and supraventricular tachycardia, but it does not have atrial fibrillation. The 200 series includes eight atrial fibrillation subjects out of 25. The 200 series also includes atrial bigeminy, atrial flutter, supraventricular tachyarrhythmia ventricular flutter and ventricular tachycardia. More detailed information about the MIT-BIH arrhythmia database can be found at <http://www.physionet.org/physiobank/database/html/mitdbdir/tables.htm>. In the preliminary work (TATENO and GLASS, 2000), we used only eight atrial fibrillation subjects from the 200 series as test data. Here, we use all the subjects of the 200 series and the 100 series.

Fig. 1 shows a typical time series of RR intervals from a patient with atrial fibrillation. The solid line represents the duration of atrial fibrillation. This line is set to atrial fibrillation when atrial fibrillation occurs; otherwise, it is set to N, which signifies a rhythm that is not atrial fibrillation. At the onset of atrial fibrillation, the rhythm dramatically changes and becomes irregular, with large fluctuations. In paroxysmal atrial fibrillation, there is sudden starting and stopping of atrial fibrillation, as indicated in Fig. 1.

ARR is defined as being the difference between two successive RR intervals. We prepared standard density histograms as a template for atrial fibrillation detection from the MIT-BIH atrial fibrillation database. Blocks of 50 successive beats were considered during atrial fibrillation in all subjects in the MIT-BIH atrial fibrillation database. Each block falls into one of 16

Table 1 Profile of MIT-BIH atrial fibrillation database

	Hours	Episodes	Beats
Atrial fibrillation	91.59	299	510293
Atrial flutter	1.27	13	10640
Other	156.12	309	700626
Total	248.98	621	1221559

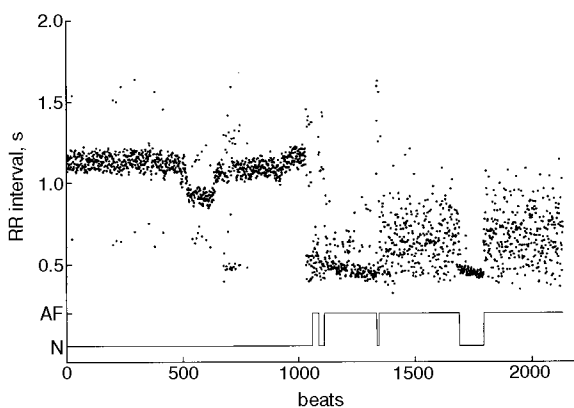


Fig. 1 Time series showing RR intervals from subject 202 from MIT-BIH arrhythmia database. (—) Assessment of atrial fibrillation (AF) or non-atrial fibrillation (N) as reported in database

different classes, identified by the mean value: 350–399 ms, 400–449 ms, 450–499 ms etc.

## 2.1 CV test

The coefficient of variation is the standard deviation of the RR intervals divided by the mean RR interval. The coefficient of variation of  $\Delta RR$  is defined to be the standard deviation of the  $\Delta RR$  intervals divided by the mean RR interval. (As the  $\Delta RR$  histograms are symmetrical and the mean value in each of the  $\Delta RR$  histograms is approximately 0, it is not useful to divide the standard deviation of the  $\Delta RR$  intervals by the mean  $\Delta RR$  interval.) As the coefficients of variation of both the RR and the  $\Delta RR$  intervals are approximately constant during atrial fibrillation, we should be able to use the coefficients of variation to detect atrial fibrillation.

The coefficients of variation of the RR and  $\Delta RR$  intervals in a test record are compared with the standard coefficients of variation to detect atrial fibrillation. The standard density histograms give us the standard coefficients of variation. To test for atrial fibrillation, we consider the 100 beat segment centred on each beat in the record and obtain the coefficient of variation of the segment. We define an acceptable range of the coefficient of variation  $R_{cv}$ . If the coefficient of variation of the test record is within the standard coefficient of variation  $\pm R_{cv}\%$ , the rhythm is labelled as atrial fibrillation. We call this the CV test.

## 2.2 Kolmogorov–Smirnov test

We compare the  $N_{seg}$  ( $= 20, 50, 100, 200$ ) beat segment centred on each beat in the record. For each beat, we determine the density histogram of the RR and  $\Delta RR$  intervals and compare these with the standard density histograms. The differences between the density histograms in a given patient and the standard histograms are evaluated using the Kolmogorov–Smirnov test (see PRESS *et al.* (1992), Section 14.3). Fig. 2 shows an example of cumulative probability distributions of the standard histogram and a test histogram.

In the Kolmogorov–Smirnov test, the greatest distance  $D$  between the cumulative probability distributions is measured. In other words, we assess whether two given distributions are different from each other. The Kolmogorov–Smirnov test returns a  $p$ -value as follows:

$$p \equiv Q(\lambda) = 2 \sum_{j=1}^{\infty} (-1)^{j-1} e^{-2j^2 \lambda^2}$$

where  $\lambda = (\sqrt{N_e} + 0.12 + 0.11/\sqrt{N_e}) * D$ .  $N_e = N_1 N_2 / (N_1 + N_2)$ .  $N_1$  is the number of data points in the standard distribution.

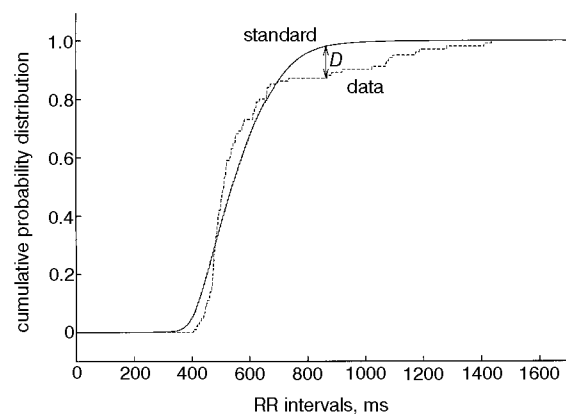


Fig. 2 Kolmogorov–Smirnov test. Distribution based on data is compared with standard distribution. Cumulative probability distribution is derived from density histogram.  $D$  = greatest distance between two cumulative distributions

$N_2$  is the number in the test distribution. Here,  $N_2$  is the segment size  $N_{seg}$ . A small  $p$ -value signifies that the distributions are significantly different from one another. As the standard density histograms represent atrial fibrillation, a value of  $p > P_c$  fails to reject the hypothesis that the test distribution is not atrial fibrillation.  $P_c$  is defined as a parameter. In short,  $p > P_c$  is associated with a positive identification of atrial fibrillation.

The results are assessed in four categories (HULLEY and CUMMING, 1988) as follows: true positive (TP): atrial fibrillation is classified as atrial fibrillation; true negative (TN): non-atrial fibrillation is classified as non-atrial fibrillation; false negative (FN): atrial fibrillation is classified as non-atrial fibrillation; false positive (FP): non-atrial fibrillation is classified as atrial fibrillation. Sensitivity and specificity are defined by  $TP/(TP + FN)$  and  $TN/(TN + FP)$ , respectively. The predictive value of a positive test (PV+) and the predictive value of a negative test (PV-) are defined by  $TP/(TP + FP)$  and  $TN/(TN + FN)$ , respectively.

The receiver operating characteristic curve gives the sensitivity, and the specificity as a parameter in the detection algorithm is varied. In this work, we vary the value of  $R_{cv}$  and  $P_c$  to determine the receiver operating characteristic curve.

### 3 Results

#### 3.1 Standard density histograms of the RR and $\Delta RR$ intervals

Fig. 3 shows the standard density histograms composed of RR intervals collected during atrial fibrillation. The standard density

histograms of RR intervals have skewed distributions, as reported by GOLDSTEIN and BARNETT (1967). Further, Figs 3g–k show bimodal distributions. Fig. 4 shows the standard density histograms composed of  $\Delta RR$  collected during atrial fibrillation. All standard density histograms of  $\Delta RR$  have symmetrical distributions. Fig. 5 shows the coefficient of variation of the RR intervals in Fig. 3 and the  $\Delta RR$  in Fig. 4. The coefficient of variation of the RR intervals is 0.24 by linear regression (see PRESS *et al.* (1992), Section 15.2). The coefficient of variation of the  $\Delta RR$  intervals is approximately constant ( $\approx 0.34$ ) during atrial fibrillation (Fig. 5).

#### 3.2 CV test

Fig. 6 shows the receiver operating characteristic curve of the CV test obtained by varying  $R_{cv}$ , and the receiver operating characteristic curve of the Kolmogorov–Smirnov test obtained by varying  $P_c$ . To test the CV test, we apply the CV test to the MIT-BIH atrial fibrillation database. Assuming  $R_{cv} = 35\%$  (coefficient of variation = 0.156–0.324), the CV test based on the RR intervals shows that the sensitivity is 86.6% and the specificity is 84.3%. An increase in  $R_{cv}$  increases the sensitivity but decreases the specificity.

A value of the receiver operating characteristic curve on a diagonal line shows the optimum value of the current test. The optimum value is where the sensitivity and the specificity are both approximately 85%. The CV test based on  $\Delta RR$  with  $R_{cv} = 35\%$  (coefficient of variation = 0.221–0.459) shows that the sensitivity is 83.9% and the specificity is 83.7%.

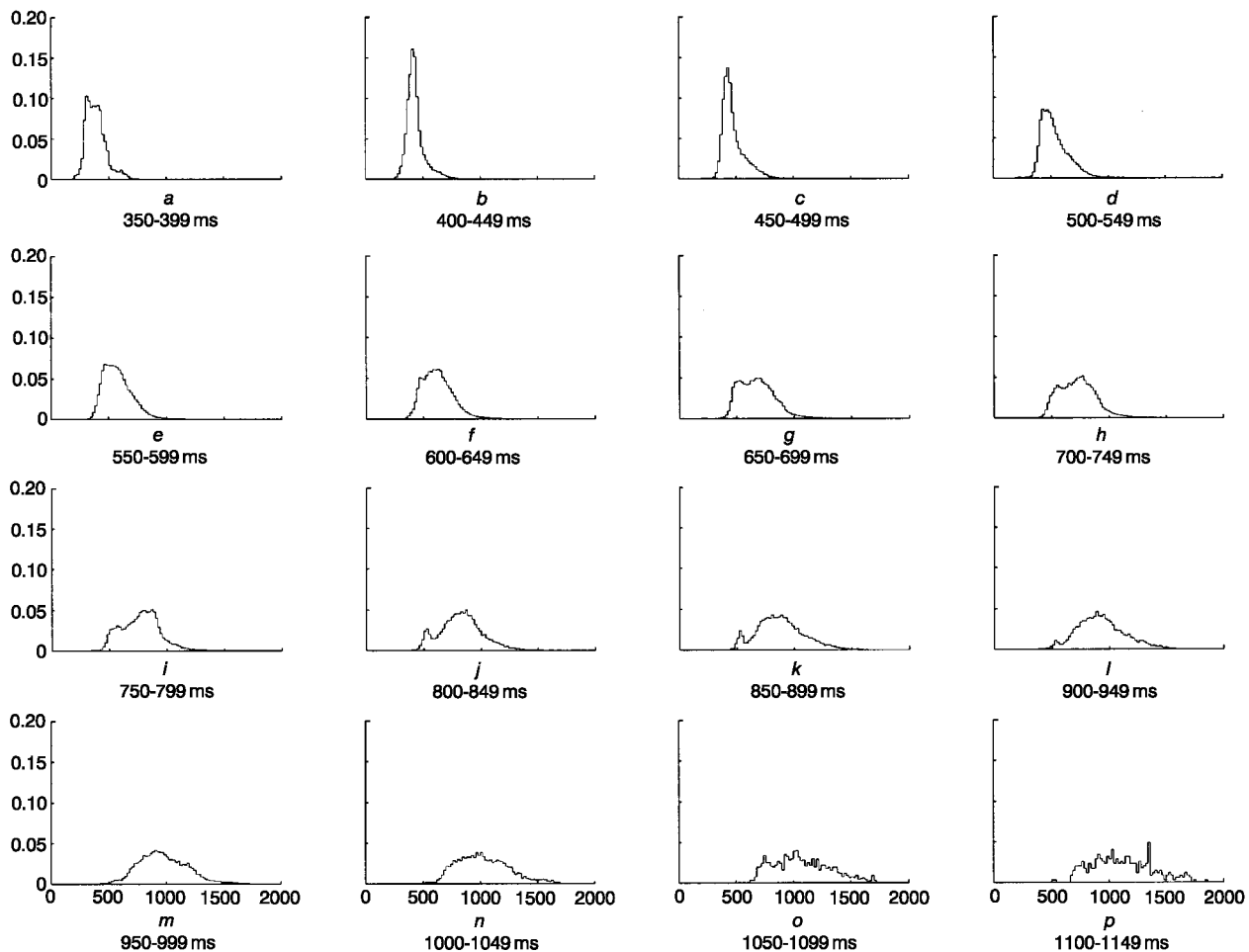


Fig. 3 Standard density histograms of RR intervals during atrial fibrillation. Time bin is 20 ms. Captions below each Figure indicate class of mean RR interval. Number of data are (a) 1900; (b) 16 250; (c) 27 400; (d) 58 950; (e) 105 700; (f) 97 700; (g) 62 800; (h) 45 650; (i) 19 300; (j) 17 100; (k) 12 800; (l) 16 550; (m) 13 250; (n) 6200; (o) 1200; (p) 350

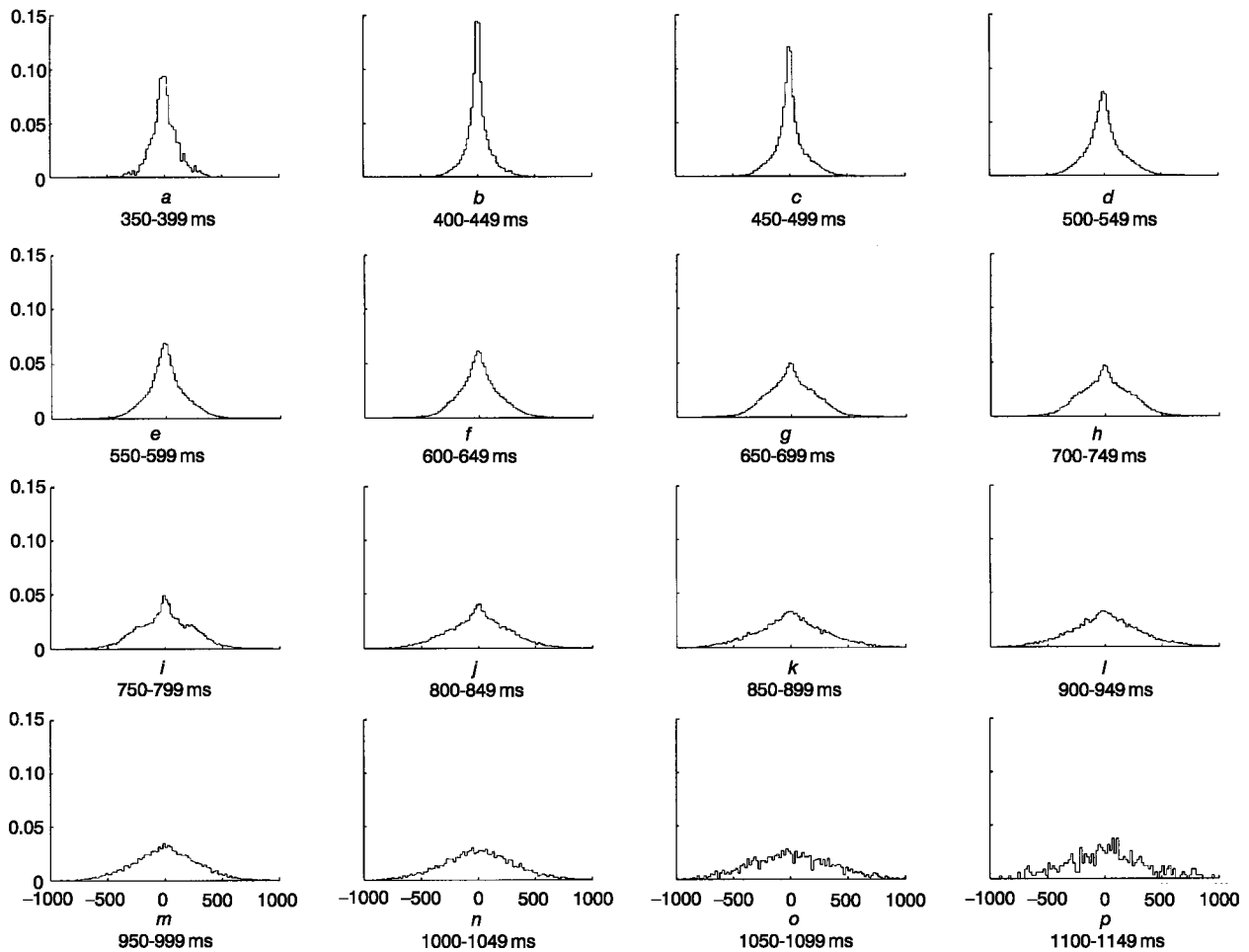


Fig. 4 Standard density histograms of  $\Delta RR$  during atrial fibrillation (for explanation, refer to Fig. 3)

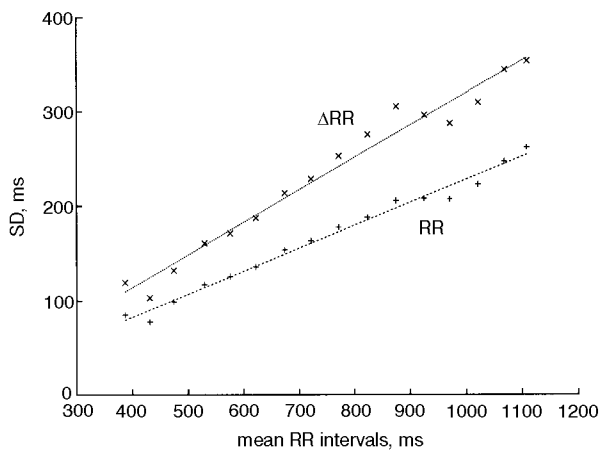


Fig. 5 Standard deviation of the standard density histograms of RR interval and  $\Delta RR$  as function of mean RR interval. Coefficient of variation of RR interval is 0.24. Coefficient of variation of  $\Delta RR$  is 0.34

### 3.3 Kolmogorov–Smirnov test

In the Kolmogorov–Smirnov test based on the RR intervals, we use the density histogram of RR intervals (Fig. 3) as the standard density histograms. Here,  $N_{seg}$  is 100. The Kolmogorov–Smirnov test based on the RR intervals shows that the sensitivity is 93.5% and the specificity is 93.6% when  $P_c = 0.000011$  (Fig. 6). An increase in  $P_c$  improves the specificity, whereas it decreases the sensitivity. To avoid increasing false positives, we choose the criterion for signifi-

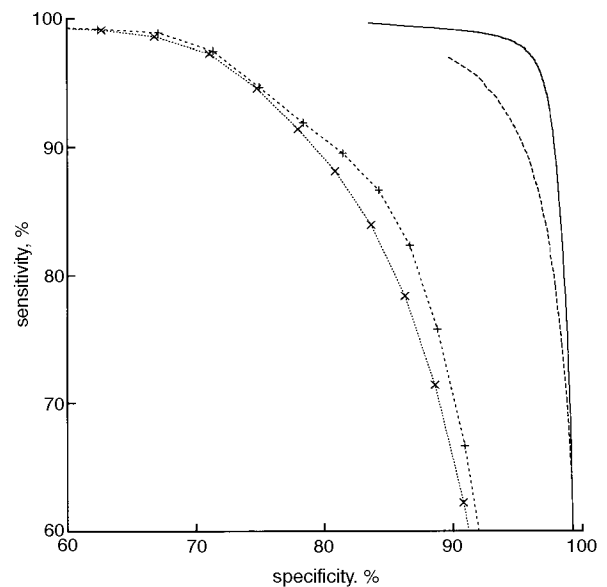


Fig. 6 Receiver operating characteristic curve for CV test and Kolmogorov–Smirnov test for MIT-BIH atrial fibrillation database. An increase in  $R_{cv}$  or decrease in  $P_c$  increases sensitivity, but decreases specificity.  $N_{seg} = 100$ . (---) KS test (RR); (—) KS test ( $\Delta RR$ ); (-+ -) CV test (RR); ( $\dots \times \dots$ ) CV test ( $\Delta RR$ )

cance as  $P_c = 0.01$ . Assuming  $P_c = 0.01$ , the Kolmogorov–Smirnov test based on the RR intervals shows 66.3% sensitivity.

In the Kolmogorov–Smirnov test based on  $\Delta RR$ , we use the density histograms of  $\Delta RR$  (Fig. 4) as the standard density

histograms. When  $P_c = 0.003944$ , the Kolmogorov–Smirnov test based on  $\Delta RR$  shows that the sensitivity is 96.5% and the specificity is 96.5% (Fig. 6). Assuming  $P_c = 0.01$ , we find that the sensitivity is 94.4%, the specificity is 97.2%, and PV+ and PV– are 96.1% and 96.0%, respectively. We summarise the assessment based on the CV test and the Kolmogorov–Smirnov test for the MIT-BIH atrial fibrillation database in Table 2. As shown in Table 2, the Kolmogorov–Smirnov test based on  $\Delta RR$  shows the best score in the current tests.

From the result of the Kolmogorov–Smirnov test based on  $\Delta RR$  with  $P_c = 0.01$ , the sensitivity and the specificity are plotted as a function of the mean RR interval class in Fig. 7. Both the sensitivity and the specificity are greater than 90% when the mean RR interval  $\geq 550$  ms. When the mean RR interval is 450 ms and 500 ms, the specificity is less than 80%. For these cases, records that were non-atrial fibrillation were frequently identified as atrial fibrillation.

The  $N_{seg}$  is varied from 100 to 20, 50 and 200 in the Kolmogorov–Smirnov test based on  $\Delta RR$ . Fig. 8 shows the receiver operating characteristic curve of the Kolmogorov–Smirnov test using different segment sizes. When  $N_{seg}$  is 20 and 50, the receiver operating characteristic curve is less steep because of an increase in false positives. As  $N_{seg}$  is small, the standard deviation of  $\Delta RR$  could be estimated as a large value, even though the rhythm is not atrial fibrillation. When  $N_{seg}$  is 200, the receiver operating characteristic curve slightly shifts to the left from the receiver operating characteristic curve using  $N_{seg} = 100$ . However, the optimum value is a sensitivity of 96.4% and a specificity of 96.4%, with  $P_c = 0.00004$ . To obtain a high value of the sensitivity and the specificity,  $P_c$  should be small.

We now apply the Kolmogorov–Smirnov test based on  $\Delta RR$  to another atrial fibrillation database that was not used to

Table 2 Accuracy of CV test and Kolmogorov–Smirnov test for MIT-BIH atrial fibrillation database ( $N_{seg} = 100$ )

	CV ( $R_{cv} = 35\%$ )		Kolmogorov–Smirnov ( $P_c = 0.01$ )	
	RR	$\Delta RR$	RR	$\Delta RR$
Sensitivity	86.6	83.9	66.3	94.4
Specificity	84.3	83.7	99.0	97.2
PV+	79.8	78.7	98.0	96.1
PV–	89.8	87.9	80.4	96.0

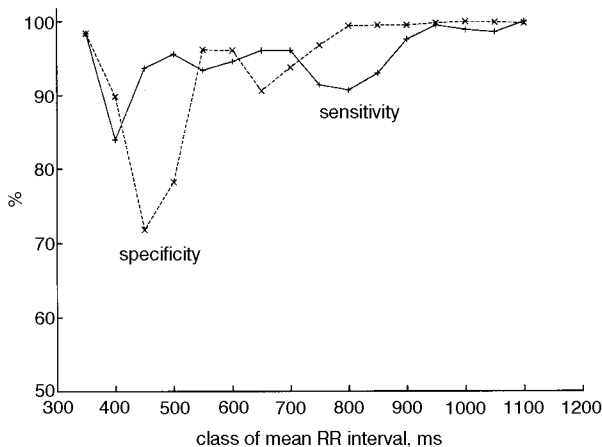


Fig. 7 Sensitivity and specificity of Kolmogorov–Smirnov test based on  $\Delta RR$  as function of mean RR interval class. Analysis was carried out on MIT-BIH atrial fibrillation database with  $P_c = 0.01$

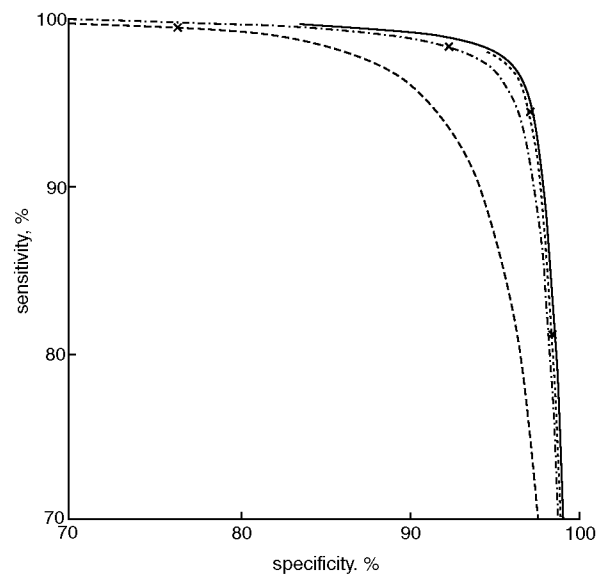


Fig. 8 Receiver operating characteristic curve depending on segment size,  $N_{seg}$ , when Kolmogorov–Smirnov test based on  $\Delta RR$  is applied to MIT-BIH atrial fibrillation database. (x) shows result when  $P_c = 0.01$ . (---)  $N_{seg} = 20$ ; (- · - ·)  $N_{seg} = 50$ ; (—)  $N_{seg} = 100$ ; (· · · ·)  $N_{seg} = 200$

construct the standard histograms. The current method is applied to the 100 series and the 200 series in the MIT-BIH arrhythmia database with  $N_{seg} = 100$ . We summarise the results in Table 3.

For the 100 series, assuming  $P_c = 0.01$ , the specificity is 99.0%. The Kolmogorov–Smirnov test based on  $\Delta RR$  clearly distinguishes atrial fibrillation from other arrhythmias. For the 200 series, the assessment of atrial fibrillation based on  $\Delta RR$  shows a sensitivity of 88.2% and a specificity of 87.6%. As shown in Table 3, the total PV+ is 62.4%. In this data set, there are difficulties identifying atrial fibrillation in three of the subjects: subjects 201, 203 and 222 show low specificity. In this data set, there are also about four times as many beats that are non-atrial fibrillation than are atrial fibrillation, and there is an increased number of false positives.

Premature ventricular contractions (PVCs) are abnormal excitations arising in the ventricles that usually lead to an abnormally short interbeat interval followed by an abnormally long interbeat interval. Although PVCs often have an altered morphology and can be detected automatically, in this study, we do not distinguish PVCs from normal QRS complexes. In the current analysis, subjects 200, 201 and 208 have frequent PVCs, and this leads to a large number of false positive identifications of atrial fibrillation in these subjects, when the Kolmogorov–Smirnov test based on  $\Delta RR$  is applied.

To illustrate the difficulties, Fig. 9 shows a cumulative probability distribution based on 100 beats collected during a segment, of subject 201, in which there are frequent PVCs. The cumulative probability distribution of the RR intervals in this subject has a prominent shoulder at about 500 ms as a consequence of the PVCs (Fig. 9a). Nevertheless, the cumulative probability distribution of the  $\Delta RR$  intervals is similar to the standard distribution observed during atrial fibrillation (Fig. 9b).

## 4 Discussion

In this paper, we have proposed a quantitative method for the determination of atrial fibrillation from the surface electro-

Table 3 Kolmogorov–Smirnov test based on  $\Delta RR$  from MIT-BIH arrhythmia database ( $P_c = 0.01$ ,  $N_{seg} = 100$ )

Subject	TP	TN	FN	FP	Subject	TP	TN	FN	FP
100	0	2221	0	0	200	0	1168	0	1381
101	0	1813	0	0	201	838	624	33	416
102	0	2135	0	0	202	723	1156	167	38
103	0	2032	0	0	203	1657	209	386	676
104	0	2177	0	0	205	0	2604	0	0
105	0	2520	0	0	207	0	2130	0	150
106	0	1949	0	26	208	0	1667	0	1236
107	0	2085	0	0	209	0	2952	0	0
108	0	1711	0	0	210	2268	0	272	58
109	0	2480	0	0	212	0	2696	0	0
111	0	2072	0	0	213	0	3199	0	0
112	0	2487	0	0	214	0	2209	0	0
113	0	1743	0	0	215	0	3311	0	0
114	0	1827	0	0	217	245	1716	93	102
115	0	1901	0	0	219	1695	228	55	124
116	0	2360	0	0	220	0	1996	0	0
117	0	1483	0	0	221	2000	4	305	66
118	0	2226	0	0	222	792	824	60	755
119	0	1935	0	0	223	0	2271	0	282
121	0	1811	0	0	228	0	1411	0	590
122	0	2424	0	0	230	0	2204	0	0
123	0	1466	0	0	231	0	1519	0	0
124	0	1567	0	0	232	0	1728	0	0
					233	0	2751	0	276
					234	0	2701	0	0
Total	0	46425	0	26	Total	10218	43278	1371	6150

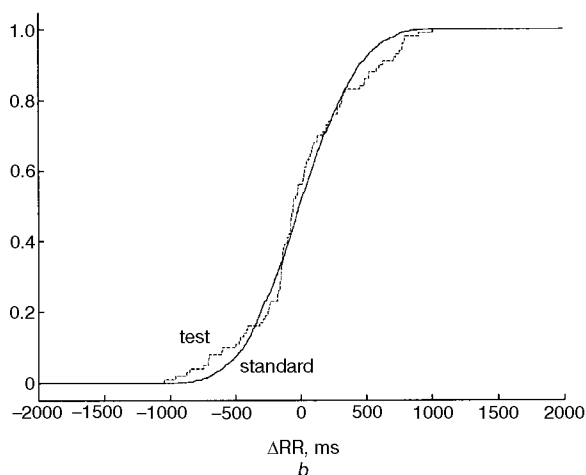
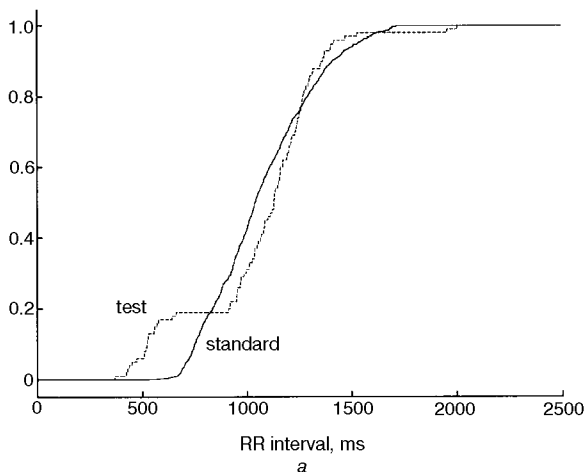


Fig. 9 Cumulative probability distribution based on 100 beats collected during rhythm with frequent PVCs compared with standard distribution. Mean value of RR intervals of segment is 1059.03 ms. Mean RR interval class is 1050–1099 ms (Fig. 3o and Fig. 4o). (a) RR interval. (b)  $\Delta RR$

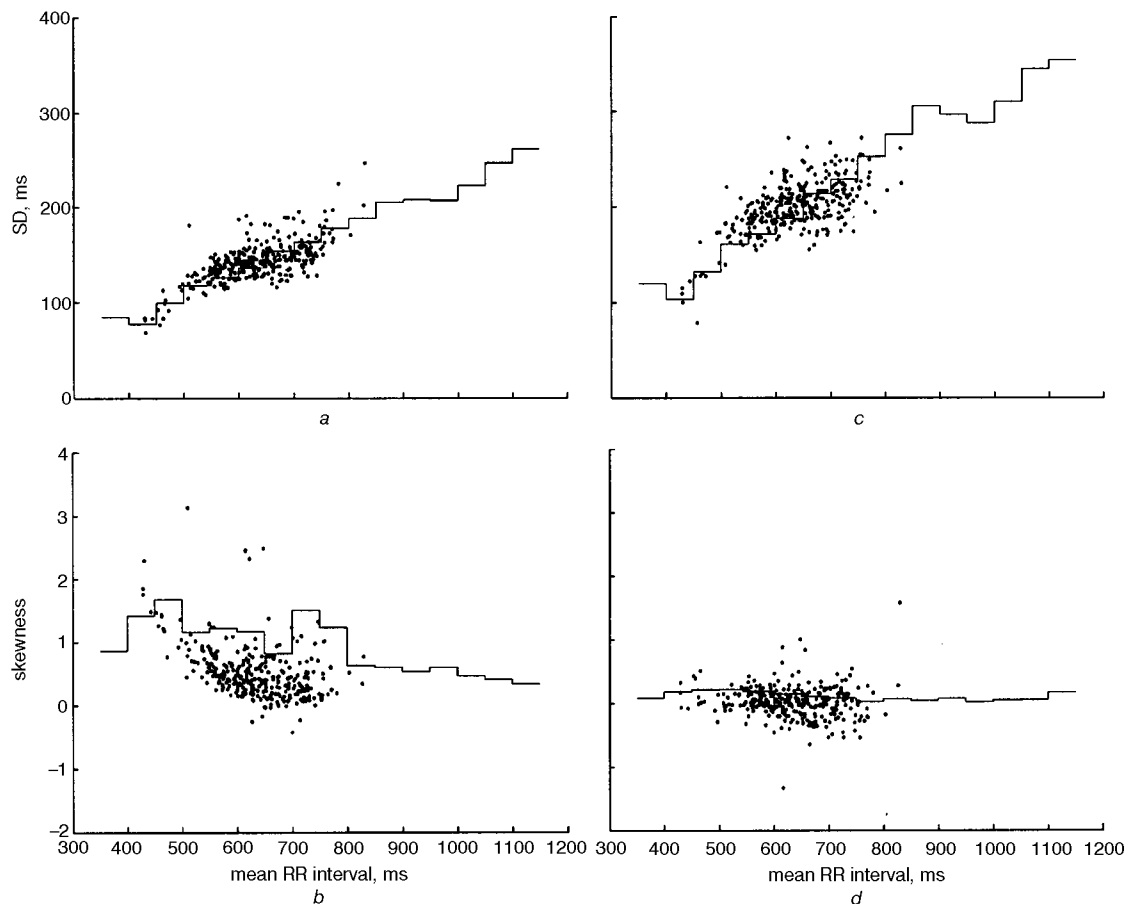
cardiogram based on the density histograms of the  $\Delta RR$  collected during atrial fibrillation.

We compared the Kolmogorov–Smirnov test with the CV test. The Kolmogorov–Smirnov test based on  $\Delta RR$  showed a sensitivity of 94.4% and a specificity of 97.2% in the MIT-BIH atrial fibrillation database. In contrast to the Kolmogorov–Smirnov test, the CV test based on  $\Delta RR$  showed that sensitivity and specificity were both approximately 84%. The Kolmogorov–Smirnov test improved the sensitivity and the specificity of the CV test. In the CV test, an increase in  $R_{cv}$  leads to false positives. During atrial fibrillation, the coefficient of variation in a test record was near the standard coefficient of variation. Some arrhythmias or transitions between arrhythmias, however, also have as large a coefficient of variation as has atrial fibrillation.

Although the Kolmogorov–Smirnov test can also be applied to the standard density histograms of the RR intervals collected during atrial fibrillation, we found that this test was not a good way to determine if the underlying rhythm was atrial fibrillation and showed low sensitivity. To assess the reason for this failure, we computed the standard deviation and the skewness of samples of 100 beats and compared these with the standard deviation and the skewness of the standard histograms collected during atrial fibrillation.

The Kolmogorov–Smirnov test is sensitive to a change in the median value of the histograms (PRESS *et al.*, 1992). During atrial fibrillation, there is variation in the shape of the RR histograms. One way to show this is to plot the standard deviation and skewness for each sample of 100 beats in a given patient as a function of RR. Figs 10a and b and 10c and d show examples of the standard deviation and the skewness of histograms of RR and  $\Delta RR$ , respectively. The standard deviation as a function of the mean RR interval falls closely along the standard values. However, the skewness in this patient falls consistently below the standard values of the histograms based on the RR distributions. The change in the skewness causes the median value to shift.

Analysis of other patients showed that the skewness of the histograms based on the RR intervals typically deviated from the



**Fig. 10** Standard deviation and skewness of density histograms of RR intervals and  $\Delta RR$  in 100 beat segment from subject 04746 in MIT-BIH atrial fibrillation database. (a) Standard deviation of density histograms of RR intervals; (b) skewness of density histograms of RR intervals; (c) standard deviation of density histograms of  $\Delta RR$ ; (d) skewness of density histograms of  $\Delta RR$ . (—) Standard deviation and skewness of standard density histogram. Standard deviation of RR interval and  $\Delta RR$  in test record falls closely along standard value. Skewness of RR interval, however, falls consistently below standard value

standard histograms and that there were, consequently, many cases in which atrial fibrillation was classified as non-atrial fibrillation. However, for the histograms based on the  $\Delta RR$  intervals, the skewness centred around 0 for both the standard and the test distributions, and the sensitivity was much improved.

A decrease in the segment size  $N_{seg}$ , keeping the value of  $P_c$  fixed, increases the number of false positives using the Kolmogorov–Smirnov test based on the  $\Delta RR$  intervals (Fig. 8). Records that are not atrial fibrillation are erroneously identified as atrial fibrillation, because the inherent fluctuations of the  $\Delta RR$  intervals in normal records can mimic the fluctuations during atrial fibrillation over short intervals of time.

The Kolmogorov–Smirnov test based on the  $\Delta RR$  intervals sometimes classifies rhythms with frequent PVCs as atrial fibrillation (Fig. 9). PVCs are often followed by a compensatory pause. Consequently, a PVC leads to a negative  $\Delta RR$  followed by a positive  $\Delta RR$ . These fluctuations in the  $\Delta RR$  intervals lead to an atrial fibrillation-like cumulative probability distribution (Fig. 9b). In contrast, the cumulative probability distributions of the RR intervals collected during rhythms with frequent PVCs have a prominent shoulder at around 400–600 ms (Fig. 9a) that could be used to help distinguish these records from atrial fibrillation. It should be possible to use the height and the width of the shoulder of the cumulative probability density distribution of the RR intervals in Fig. 9a to distinguish records with frequent PVCs from atrial fibrillation.

Preliminary studies using the Kolmogorov–Smirnov test based on the  $\Delta RR$  intervals applied to the 200 series of the

MIT-BIH arrhythmia database showed that a reduction in the false positives of approximately 6% could be achieved by classifying records with a prominent shoulder in the density histograms of the RR intervals at approximately 400–600 ms as records with frequent PVCs rather than atrial fibrillation.

This work adopts a different strategy for atrial fibrillation detection from the earlier proposal by MOODY and MARK (1983). Moody and Mark use a Markov model in which the probabilities for transitions between short, regular and long RR intervals of a test record are compared with the transition probabilities measured during atrial fibrillation. In this test, transitions from a regular interval to a short interval have a high weight in identifying the rhythm as atrial fibrillation. As rhythms with frequent PVCs have many such transitions, there is a tendency to identify these rhythms as atrial fibrillation. Consequently, although the Markov model has as high a sensitivity as that of the Kolmogorov–Smirnov test, the Markov model tends to have a relatively low predictive value of a positive test (PV+). Applying the Markov model to the 200 series of the MIT-BIH arrhythmia database, we find that the sensitivity was 87.3% and the predictive value of the positive response was 46.2%.

An interesting feature of atrial fibrillation that is poorly understood is the relative constancy of the coefficient of variation. In earlier work, Meijler *et al.* developed a model for AV nodal conduction that assumed that the constancy of the coefficient of variation was associated with concealed conduction in the AV node (MEIJLER *et al.*, 1996; MEIJLER and WITTKAMPF, 1997). The current work does not give indication

of the mechanism. However, it will be of great interest to compare histograms of atrial and ventricular intervals for different mean values of ventricular response. It should be possible to determine the relative contributions of autonomic tone and atrial activity to the timing of ventricular activation. Meijler *et al.* hypothesise that, owing to concealed conduction, rapid activation of the atria leads to a paradoxical increase in the ventricular interbeat intervals (MEIJLER *et al.*, 1996).

An accurate automatic detector of atrial fibrillation would be useful clinically. Following a successful cardioversion to restore normal sinus rhythm, an automatic monitor could be used to monitor the relapse of atrial fibrillation. For patients with paroxysmal atrial fibrillation, it could provide a good way to assess the relative lengths of time the patient is in atrial fibrillation and non-atrial fibrillation and it therefore could be useful in monitoring the efficacy of anti-arrhythmic drugs.

In summary, as the coefficients of variation of both the RR and  $\Delta$ RR intervals are approximately constant during atrial fibrillation, this can provide a basis for testing for atrial fibrillation. However, an improved method for testing for atrial fibrillation compares the histograms of  $\Delta$ RR intervals during atrial fibrillation with standard density histograms.

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## Authors' biographies

KATSUMI TATENO obtained his PhD in Electrical Engineering from Kyushu Institute of Technology in Fukuoka, Japan, in 1999, studying non-linear dynamics of neural networks. He is currently a Postdoctoral Fellow in the Department of Physiology at McGill University studying complex cardiac rhythms in clinical data and *in vitro* experiments.

LEON GLASS obtained his PhD in Chemistry in 1968 from the University of Chicago. He was then a Postdoctoral Fellow in Machine Intelligence and Perception (University of Edinburgh), Theoretical Biology (University of Chicago) and Physics and Astronomy (University of Rochester). He has been at McGill University, Montreal, Canada, since 1975, where he is now the Isadore Rosenfeld Chair in Cardiology and a Professor of Physiology. He has been a Visiting Professor at the University of California at San Diego (1984–85). In 1994–1995, he was a recipient of a Guggenheim Fellowship to study nonlinear dynamics and sudden cardiac death at Harvard Medical School.