Biology 309a: Problem Assignment #4

Due Tuesday December 6, 1994

Problems A, B, C

Book Problems 5.4, 5.16, 5.9

Problem D

Sketch flows that: 1) Produce a limit cycle. 2) Produce two limit cycles. 3) Produce a figure-8. (Hint: it's not possible. Explain why.)

Problem E

You are an astrophysicist/cosmologist, studying the origin of the universe. You invent a theory that the production of neutrinos in a black hole is coupled to radial oscillations in the Schwarzschild radius according to the following equation, which holds close to the black hole's steady state:

$$\begin{aligned} \frac{dx}{dt} &= Ax - By \qquad \text{Neutrinos} \\ \frac{dy}{dt} &= Cx + Dy \qquad \text{Schwarzschild radius.} \end{aligned}$$

Unfortunately, you have no way to measure the parameters A, B, C, and D. In order to publish your results and receive fame and fortune, you decide to personalize the black-hole dynamics and assign the parameters in the following way:

A The last digit of your phone number (e.g., 7).

B The number of letters in your first name.

C The number of letters in your last name.

D Your age in years.

Find the eigenvalues and say whether the fixed point at $x = 0$, $y = 0$ is stable and whether there are oscillations around the fixed point.

From your calculations, you discover that the Schwarzschild radius is growing exponentially with time. This is bad news, because anything within the radius will be swept into the black hole. A New York *Times* reporter asks you to put a positive slant on the story.

Explain the limitations of the eigenvalue analysis of the neutrino-Schwarzschild interaction, and why you can be confident in the form of the equation, even without knowing A, B, C, and D.

Bonus

In the Lotka-Volterra equations, the predator population is given by

$$\frac{dy}{dt} = \gamma xy - \delta y.$$

This means that in the absence of prey (i.e., $x = 0$), the predator dynamics are exponential decay to zero.

Re-write the predator equation so that in the absence of prey, the predator population is governed by Verhulst growth.

Find the fixed points of the system that correspond to extinction of the prey. Show that one of these fixed points is stable.