Fall 1999 Course 513-697: Applied Linear Models

Highlights / Key Concepts in NKNW4 Chapter 1

- Statistical Relationship/Pattern in observations made at different values of an 'explanatory' variable X
  - Systematic variation in "Centre" of all possible "responses" observed at X -- as a function of X
    - ("centre" doesn't necessarily have to be the mean; it could be the median or another centrality parameter)
    - (nor does the systematic variation have to be linear in X)
  - Random Variation around these "centres" (again, nothing specific about distribution of Y at each value of X)

- Origins of the term "regression" : Galton and his observations on "regression to mediocrity"
  (our use of the term regression has little to do with this original context)

- Statistical Model(s) : Focus on 2 components:
  - pattern of the "Centers" of Y | X at each (possible) value of X ... form of this pattern / signal
  - spread of Y | X at each (possible) value of X (Noise") (can fit models without strong assumptions about noise)
  - the term "error" is misleading; biologic and other variations are not "errors" in sense of "experimental mistakes"

- Uses of Regression Models
  - description / summarization (can think of as descriptive statistics)
  - prediction
  - poor person's experimental control .. multiple regression: adjustment for "confounding" variables

- SIMPLE LINEAR REGRESSION MODEL
  - E{Y | X} linear in X and linear in regression parameters
  - Pattern/Distribution of spread of { Y | X} unspecified

- Meaning of regression parameters β_0 and β_1
  - Think of β_1 as ∂ E{Y | X} / ∂X - β_0 usually of less interest (or maybe even completely artificial)

- Alternative ("Centered") Form for E{ Y | X} : $\beta_0^* + \beta_1[X - \bar{X}]$
  - Think of β_1 as ∂ E{Y | X} / ∂X - β_0 usually of less interest (or maybe even completely artificial)

- Data
  - X values can be stochastic or selected (efficiencies possible if allowed to select which X values to study)
  - X values assumed to be measured without error

- Parameter Estimation
  - different fitting criteria Least Squares; Least Absolute Deviations; Least Perpendicular Deviations; ...
  - Maximum Likelihood estimation possible if specify a statistical distribution for spread of Y | X

- Method of Least Squares (LS)
  - assumptions (few!)
  - derivation of computational formulae for estimators for β_0 , β_1
  - Properties of LS estimators. of residuals, and of fitted line

- Estimation of Var{ Y | X} ... assuming that Var{ Y | X} is constant (homogeneous) over range of X
  - via Mean Square Error (MSE)

- Normal (Gaussian) Error Regression Model
  - assumptions
  - derivation of ML estimators for β_0 , β_1 and for ML estimators for Var{ Y | X}
  - Equivalence of LS and ML estimators for β_0 , β_1 when assume Normal (Gaussian) Error Regression Model