Q 5.18 (a)

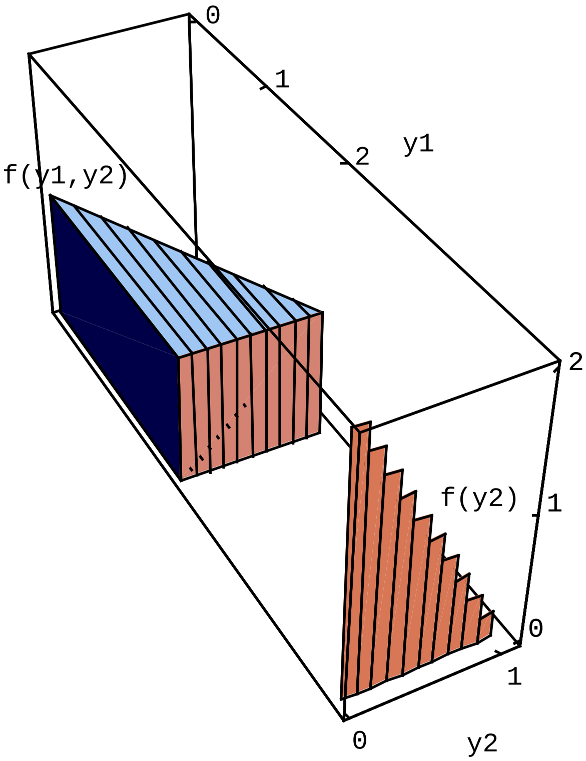
Question is about the marginal distribution of Y2. So, 1st, we must derive f(y2)

Joint distribution is shown as a constant density of 1 over top of the triangle bounded by

$y_1 = 2$, $y_2 = 0$, and $y_1 = 2\, y_2$.

Marginal distribution of Y_2 is obtained by "collapsing", "aggregating over", or summing (integrating) over Y_1. For illustration, we discretize Y_2 into 10 intervals, so that the volumes of the various 0.1 units thick slices are shown, and then "transferred" to the margin.

[I left a large space between the joint and marginal distributions so we could "see" each one clearly.. In fact, where (if anywhere) on the "Y_1" axis we choose to situate the Y_2 distribution is arbitrary - after all it is the distribution of Y_2 "regardless" of Y_1.



Now that we have $f_2(y_2)$, we treat it like any other density..

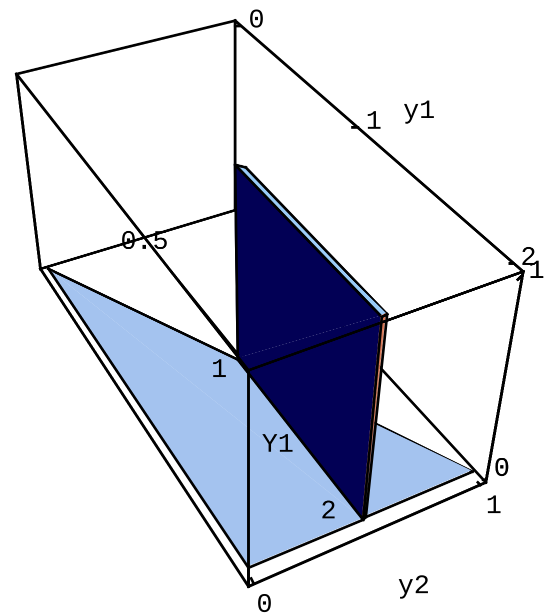
$$f_2(y_2) = \begin{cases} 2 - 2y_2 & \text{on } (0,1), \\ 0 & \text{elsewhere.} \end{cases}$$

First, check that this density integrates to unity. Its base is 1 and height is 2, so it does.

Then we can find $P(Y_2 > 0.5)$ by integrating $f_2(y_2)$ over the interval (0.5,1). From the geometry, this area is (1/2) x 0.5 x 1 = 1/4.

Q 5.18 (b) This question concerns a particular CONDITIONAL distribution of Y_1, that is, conditional on Y_2 being equal to 0.5

[Note: in fact, with continuous Y_2, there is zero probability that Y_2 will be exactly 0.5, or any other "exact" value for that matter... So, instead, let us take out the "very thin" slice of probability on each side of Y_2=0.5.



It so happens that the marginal density f_2, of Y_2 at y_2=0.5 is 1, so that the re-scaling of the dark blue slice of probability is not necessary.. its surface area is already unity.

i.e. $$f_1(\, y_1 \mid Y_2 = 0.5) = \frac{f(y_1, 0.5)}{f_2(0.5)}$$

Had we taken a slice at Y_2=0.3, the rectangle would have had a height of 1, but a base of 1.4, so its area would be greater than unity (and so not a density)

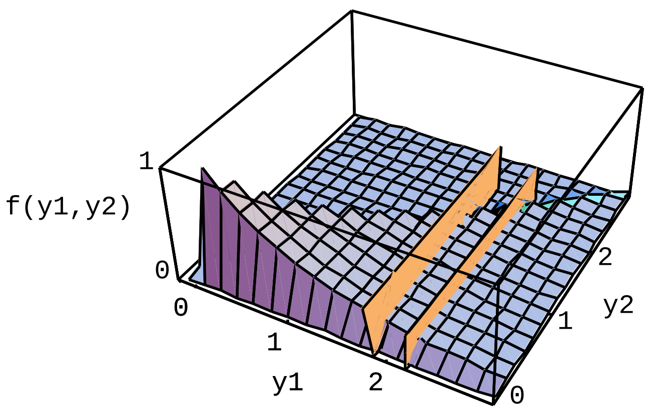
The scaling would have been {see text, p205)

$$f_1(\, y_1 \mid Y_2 = 0.3) = \frac{f(y_1, 0.3)}{f_2(0.3)} = \frac{f(y_1, 0.3)}{1.4}$$

Q 5.21 (see 5.9)

Question is about the conditional distribution of Y_2, given Y_1 has the specific value y_2=2.0

Joint distribution is positive in half of the positive quadrant...



Let us "extract" the slice at Y_1 = 2.0...

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Plot[(f[y1,y2]/.y1->2.0),{y2,-1,3},
AxesLabel->{"y2","f(y1,y2) at y1=2"},
Ticks->{{0,1,2},{0,.05,.1}}]
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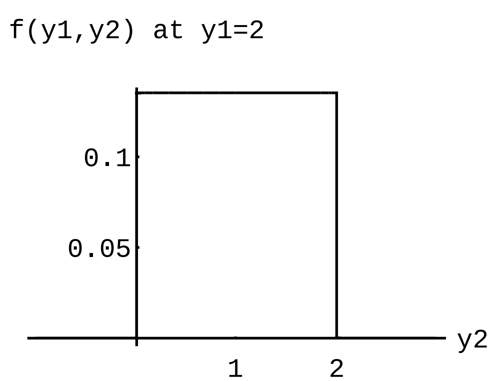


Notice that this function cannot serve as a density function, since the area under it is not unity: its base is 2 and its height is exp(-2)=0.14, yielding an area of only 0.28. But multiplying the height by 1/0.14 does make it a legitimate probability density funbction, uniform on (0,2).

the divisor, 0.14, is the marginal probability that Y_1 = 2.0

See text definition 5.7, page 205

Obvious from joint density that, marginally, Y_1 has an exponential density $f_1(y_1)$ = exp($-y_1$) on (0,infinity)

If in doubt, integrate the joint density $f(y_1,y_2)$ over Y_2

Q 5.36 (see 5.8)

The bivariate pdf equals 2 over the triangular region

$0 \le y_1 \le 1$,
$0 \le y_2 \le 1$
$0 \le y_1 + y_2 \le 1$

Are Y_1 and Y_2 independent?

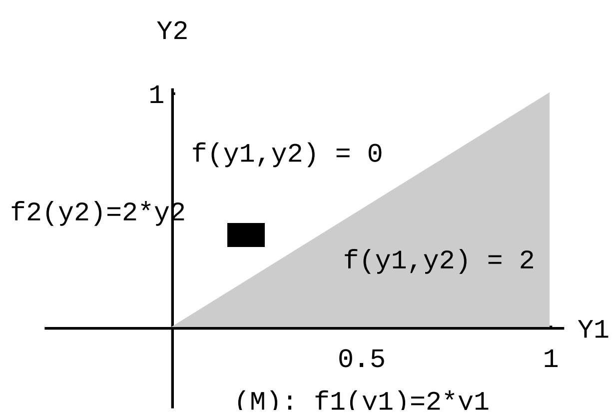
Answer: Clearly not!

If they were independent, the density at a particular (y_1,y_2) point would be the product of the marginal density functions evaluated at y_1 and y_2 respectively. As can be seen, the marginal probability function for y_1 is everywhere positive in the interval $0 \le y_1 \le 1$, and likewise for y_2 over (0,1)

But all of the (y_1,y_2) pairs above the diagonal have probability zero, whereas they would have been predicted to have positive probability under independence

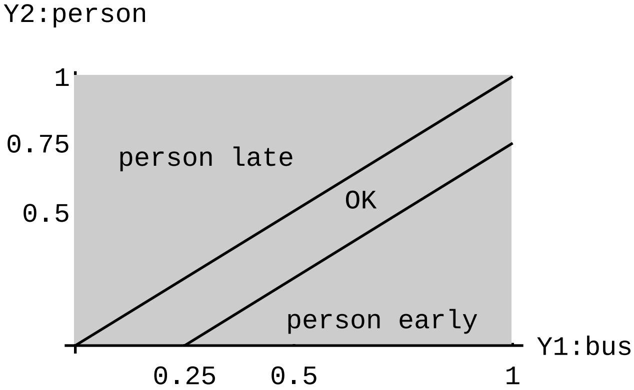
Q 5.37 (see 5.9)

The bivariate pdf is positive over the triangular region below the diagonal, and zero above it (for example, zero mass over the black rectangular region). The marginal probabilities (M) are everywhere positive, and so their product cannot be zero..
So, Y_1 and Y_2 are not independent



Other sign: $f(y_1,y_2)$ does not factor!

Q 5.42 (bus)



Probability of person catching the bus is obtained by integrating f(y1,y2)=1 over the "OK" region..

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or as 1 - [ prob (early)   + prob(late) ]

    = 1 - [(1/2)(3/4)(3/4) + 1/2        ]

    = 1 - [    9/32        + 16/32      ]

    = 1 - [ 25/36 ] = 7/32
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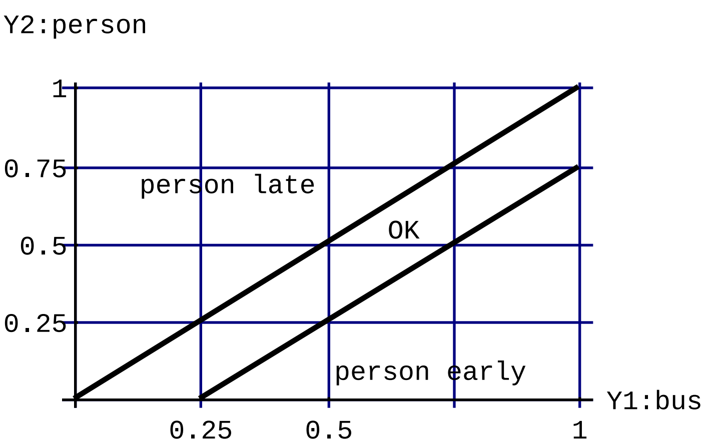
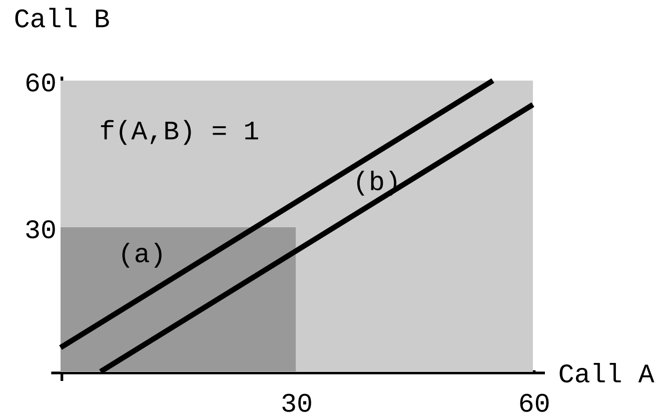
or count (7) triangles, each of area 1/32



Q 5.42 (2 indep. telephone calls)



Prob(within 5 min of ech other) = total probability mass over region (b)

approx: (10 x 60) / (60 x 60) = 1/6

exact: small correction for the ends..
book says 23/144 ..looks right

Prob(both in 1st 30 min)

= probability mass over region (a)

= 1/4

= Prob (A in 1st 30 min)
x
Prob (B in 1st 30 min)

Since A and B arrive independently.