A finely stratified log-rank test with effectively-infinite-size comparison groups

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Abstract

This presentation will elaborate on a previously published longevity analysis of ours. It will discuss some technical statistical issues related to how we created synthetic comparison groups, and how we avoided the errors made by other authors, e.g., those who have studied the longevity of jazz musicians. The differences between 'current lifetables' – commonly used, and frequently misunderstood by the public – and cohort lifetables will be illustrated. The synthetic nature of the comparison groups we created is used to illustrate the limiting behaviour of the stratified log rank test when each stratum consists of one index person and an 'effectively infinite' number of comparison persons.

Keywords: Survival Analysis, Lexis Diagram, Current and Cohort Life-Tables, Log-rank Test.

1 Introduction

Failure to recognize what is now termed "immortal time bias" has, over the years, lead to a large number of invalid survival comparisons in the medical and epidemiologic literature (Hanley et al. 2006). In teaching how to perform more valid analyses, we have used as an example the longevity of the passengers who survived the sinking of the Titanic (Hanley et al. 2003). We compared the post-1912 longevity of each passenger with the remaining life-course of an age- and sex-matched group of peers alive in 1912, using each (passenger, peer-group) as a separate 'stratum.' Since each comparison lifetable was reconstructed from national vital statistics data, the comparison group for each passenger was effectively infinite in size.

2 Methods

Unable to find a comparison group with the same mix of backgrounds and selection factors, we created two 'next best' comparison groups from available national data. We calculated what proportions of an age and sex matched group of white Americans alive in 1912 would be alive at each anniversary of the sinking. To do so, we converted current (cross sectional) life tables for the years 1912-20002 into cohort life tables. The Lexis Diagram helped guide the calculations. We created a second comparison group from life table data for Sweden, which were already in cohort form. Longevity differences were assessed by several methods, including a stratified log rank test.

3 Results

Substantively, the survival of the 435 passengers who had been traced was slightly, but not significantly, longer than that of the two comparison groups. Despite their social advantage, first class passengers did not fare particularly well.

Methodologically, if we denote by t the age at death of a passenger, and by $S[t]$ the corresponding proportion in the comparison population still alive at that age, then the stratified Log-Rank statistic (with $1df$) has the simple form $\{\sum(1+\log S[t])\}^2/\{-\sum \log S[t]\}$ where $\sum$ denotes summation over the n pasengers. Alternatively, if we combine the passenger-specific P-values, we obtain the Chi-squared ($2n$ df) statistic $-2\sum \log S[t]$.

4 Discussion

Given our inability to find a comparison group with the same mix of backgrounds and selection factors, the inaccuracies in the data, and the fact that some 17% have not been traced, we do not wish to over-interpret the substantive results. However, the special nature of this particular stratified log-rank test, when each stratum consists of one person in the index category, and an infinite number in the reference category, does lead to some insights into the structure of the limiting case of the log-rank test, and into the name of the test itself.

References

Hanley JA, Carrieri MP and Serraino D (2006). *International Journal of Epidemiology* Advance Access published March 16, 2006

Hanley JA, et al. (2003). *BMJ* 327 (7429):1457-8.