Suggested Exercises from M \& M Chapter 6 Homegrown exercises begin on page 2

These pages were updated on September 30
To start with, do some of the odd-numbered exercises. answers to all odd-numbered exercises are given on textbook pages S-1 onwards.

Do some or all of the following even-numbered exercises. You are asked to hand in answers to designated ones.. see the list, and the deadline, on the main course page. Some of these will be discussed in tutorials or answers to them posted on the course web page

| $\S 6.1$ | $\S 6.2$ | $\S 6.3$ |
| :--- | :--- | :--- |
|  |  |  |
| 6.4 | 6.26 | 6.53 |
| 6.6 | 6.28 | 6.54 |
| 6.12 | 6.30 | 6.55 |
| 6.17 | 6.32 | 6.58 |
| 6.18 | 6.34 | 6.59 |
| 6.20 | 6.38 | 6.62 |
| 6.22 | 6.40 |  |
| 6.24 | 6.48 |  |
| 6.76 | 6.50 |  |
| 6.82 a | 6.52 |  |
|  | 6.84 |  |
|  | 6.85 |  |

## "Homegrown" Exercises around M \& M Chapter 6

## -1- Help a journalist to be "statistically correct"

See -- under resources for Chapter 6 -- the excerpt from the article <<Controverse autour des pesticides comme agents du cancer du sein>> by M Perreault, La Presse, Montreal, Jeudi 29 Juillet 1999.

After reporting that a finding was 'not statistically significant', the journalist goes on to explain what 'statistically significant' means. For those who need it, here is my translation [with approximately $85 \%$ confidence!] of what was stated
"In general, an average increase in risk is valid if 95\% of the data show a higher risk than in the control group; in other words, the results can be reproduced 19 times out of $20^{\prime \prime}$

Rewrite this to explain
a a reported relative risk which has an associated "P-value of 0.03 "
b a "95\% Confidence Interval" accompanying the reported relative risk.

## -2- Handedness and Mortality: A Follow-Up Study of Danish Twins Born between 1900 and 1910

Olga Basso, Jørn Olsen, Niels V. Holm, Axel Skytthe, James W. Vaupel, and Kaare Christensen

Epidemiology vol 11 no 5 sept 2000
The declining prevalence of left-handed individuals with increasing age has led to two main avenues of hypotheses; the association is due either (1) to a birth cohort effect and/or an age effect caused by a switch to right-handedness with advancing age or (2) to mortality selection that reduces survival in left-handed individuals, or both. It is uncertain whether a cohort or age effect can explain the decline in
age-related prevalence, and conflicting evidence exists in favor of the mortality hypothesis. We compared mortality in a subgroup of 118 opposite-handed twin pairs by counting in how many instances the right-handed twin died first. There was no evidence of differential survival between right-handed and non-right-handed individuals in the entire 1900-1910 cohort. With respect to the number of righthanded twins who died first, there was no material disadvantage among those who were not right-handed. In $60 \%$ ( $95 \%$ confidence interval $=49.0-71.5 \%)$ of dizygotic pairs, the right-handed twins
died firs ${ }^{\text {a }}$ In $50 \%$ of monozygotic pairs ${ }^{-}$, right-handed twins died first. The prevalence of not being right-handed was higher among males ( $9.2 \%$ ) than females ( $6.5 \%$ ); there was a similar frequency of non-right-handedness in monozygotic ( $8.0 \%$ ) and dizygotic ( $7.8 \%$ ) twins. We did not find evidence of excess mortality among non-right-handed adult twins in this follow-up study.

Key words: mortality, survival, handedness, twin studies.
a (Approximately) how many dizygotic twin pairs must there have been?
b (Approximately) what is the corresponding CI to accompany the estimate of $50 \%$ calculated from monozygotic pairs?
c Is the $60 \%$ significantly different from the $50 \%$ at the "conventional" significance level ( $\mathrm{P}<0.05$ )?
d Calculate the percentage -- of the overall 118 twins pairs -- where the right-handed twin died first, along with an accompanying 95\% CI.

## DISTINGUISHING POPULATIONS WITH DIFFERENT MEAN BIRTHWEIGHTS

The entries in the 4 panels below represent birthweights, recorded to the nearest 10 grams, but with the ending 0 removed to save space. Thus the very first entry of 336 in Panel A represents a birthweight of 3360 grams or 3.36 Kg . The birthweights in a panel are all from infants of the same sex, but different panels may be from different sexes. The standard deviation of the entries in each panel is approximately $\mathrm{SD}=43$ (430 grams).

By eye, by comparing all the entries in a panel with all of those in another, you may be able to discern if two panels have different means. But what can you conclude if you take just a sample from each of 2 panels and perform a formal test of significance on the difference in the sample means? Details for exercise are explained on $\mathbf{p} 5$.

| PANEL A |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 336 | 357 | 338 | 379 | 386 | 362 | 277 | 340 | 404 | 300 |
| 295 | 340 | 264 | 317 | 303 | 342 | 340 | 400 | 348 | 327 |
| 294 | 390 | 347 | 346 | 294 | 407 | 408 | 380 | 343 | 413 |
| 346 | 360 | 321 | 379 | 338 | 345 | 377 | 362 | 318 | 341 |
| 428 | 346 | 354 | 358 | 353 | 401 | 338 | 283 | 356 | 275 |
| 366 | 303 | 351 | 378 | 413 | 381 | 319 | 312 | 298 | 281 |
| 372 | 380 | 282 | 303 | 345 | 282 | 445 | 304 | 339 | 357 |
| 314 | 264 | 380 | 389 | 264 | 325 | 327 | 298 | 334 | 347 |
| 299 | 428 | 338 | 277 | 268 | 310 | 345 | 316 | 396 | 381 |
| 400 | 318 | 341 | 321 | 328 | 370 | 336 | 371 | 371 | 449 |


| $\mathbf{P A N E L} \mathbf{B}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 397 | 399 | 306 | 371 | 356 | 368 | 362 | 396 | 338 | 326 |
| 331 | 411 | 422 | 413 | 381 | 399 | 385 | 333 | 293 | 311 |
| 319 | 349 | 268 | 383 | 398 | 328 | 385 | 373 | 274 | 467 |
| 328 | 377 | 300 | 341 | 386 | 387 | 265 | 411 | 378 | 358 |
| 373 | 336 | 366 | 325 | 322 | 283 | 329 | 323 | 327 | 401 |
| 292 | 313 | 340 | 424 | 311 | 363 | 335 | 350 | 343 | 364 |
| 348 | 298 | 314 | 401 | 384 | 362 | 370 | 375 | 373 | 312 |
| 399 | 355 | 435 | 437 | 362 | 316 | 371 | 340 | 315 | 359 |
| 414 | 302 | 317 | 407 | 432 | 334 | 428 | 386 | 406 | 388 |
| 325 | 334 | 448 | 344 | 373 | 296 | 301 | 347 | 361 | 294 |
|  |  |  |  |  |  |  |  |  |  |

## PANEL C

| 344 | 382 | 358 | 429 | 398 | 336 | 406 | 366 | 385 | 357 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 258 | 346 | 401 | 315 | 430 | 373 | 377 | 346 | 378 | 357 |
| 346 | 406 | 425 | 346 | 367 | 347 | 388 | 348 | 300 | 326 |
| 333 | 397 | 355 | 282 | 360 | 421 | 416 | 346 | 370 | 329 |
| 366 | 360 | 282 | 393 | 329 | 352 | 450 | 371 | 379 | 323 |
| 430 | 397 | 349 | 321 | 334 | 369 | 367 | 274 | 427 | 355 |
| 349 | 393 | 295 | 372 | 283 | 313 | 316 | 268 | 334 | 413 |
| 322 | 397 | 309 | 348 | 376 | 345 | 497 | 343 | 361 | 391 |
| 327 | 374 | 344 | 354 | 322 | 277 | 287 | 396 | 323 | 389 |
| 391 | 303 | 319 | 314 | 368 | 389 | 343 | 342 | 330 | 369 |


| $\mathbf{P A N E L} \mathbf{D}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 262 | 328 | 363 | 399 | 328 | 375 | 310 | 417 | 278 | 346 |
| 340 | 350 | 364 | 299 | 318 | 339 | 307 | 381 | 314 | 388 |
| 355 | 290 | 331 | 304 | 351 | 333 | 382 | 310 | 331 | 287 |
| 370 | 356 | 394 | 265 | 368 | 288 | 448 | 416 | 350 | 333 |
| 306 | 360 | 236 | 273 | 381 | 435 | 332 | 323 | 349 | 354 |
| 294 | 337 | 390 | 408 | 299 | 345 | 375 | 428 | 273 | 353 |
| 407 | 419 | 333 | 331 | 330 | 387 | 303 | 275 | 334 | 335 |
| 391 | 348 | 348 | 302 | 356 | 370 | 374 | 353 | 352 | 432 |
| 353 | 346 | 356 | 342 | 382 | 293 | 348 | 332 | 375 | 350 |
| 346 | 407 | 339 | 364 | 288 | 389 | 282 | 434 | 380 | 378 |
|  |  |  |  |  |  |  |  |  |  |

Key
Cailíní[céad/deireadh -- trí céad, daiched is a trí/seacht]
Buachaillí [-- trí céad, deich is daichead, is a sé]

The entries in the 4 panels below represent adult heights, recorded to the nearest centimetre. Thus the 1st entry (188) in Panel A represents a height of 188 cm or 1.68 m . The birthweights in a panel are all from adults of the same sex, but different panels may be from different sexes. The standard deviation of the entries in each panel is approximately SD $=6 \mathrm{~cm}$.

By eye, by comparing all the entries in a panel with all of those in another, you may be able to discern if two panels have different means. But what can you conclude if you take just a sample from each of 2 panels and perform a formal test of significance on the difference in the sample means? Details for exercise are explained on $\mathbf{p} 5$.

| PANEL A |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 188 | 178 | 175 | 168 | 169 | 171 | 170 | 166 | 161 | 171 |
| 180 | 178 | 184 | 174 | 168 | 176 | 175 | 167 | 182 | 177 |
| 181 | 183 | 185 | 178 | 165 | 172 | 178 | 176 | 164 | 186 |
| 176 | 179 | 169 | 169 | 184 | 169 | 173 | 173 | 173 | 177 |
| 177 | 170 | 179 | 183 | 183 | 172 | 189 | 181 | 174 | 171 |
| 170 | 182 | 163 | 171 | 176 | 176 | 183 | 181 | 174 | 175 |
| 171 | 167 | 175 | 175 | 174 | 168 | 170 | 175 | 185 | 181 |
| 183 | 180 | 178 | 170 | 174 | 173 | 176 | 173 | 175 | 173 |
| 165 | 172 | 175 | 183 | 167 | 171 | 176 | 182 | 174 | 170 |
| 187 | 185 | 167 | 169 | 168 | 178 | 182 | 178 | 171 | 175 |


| $\mathbf{P A N E L} \mathbf{B}$ |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 156 | 159 | 169 | 161 | 157 | 158 | 171 | 166 | 169 | 170 |
| 168 | 170 | 175 | 171 | 167 | 168 | 160 | 170 | 173 | 165 |
| 160 | 162 | 156 | 150 | 168 | 157 | 168 | 167 | 159 | 168 |
| 159 | 165 | 165 | 165 | 164 | 163 | 159 | 169 | 176 | 176 |
| 166 | 155 | 164 | 162 | 172 | 172 | 156 | 166 | 166 | 161 |
| 165 | 162 | 177 | 162 | 160 | 171 | 164 | 174 | 164 | 173 |
| 174 | 160 | 164 | 163 | 171 | 172 | 159 | 157 | 159 | 168 |
| 161 | 166 | 160 | 167 | 168 | 162 | 158 | 154 | 159 | 167 |
| 166 | 163 | 166 | 177 | 168 | 172 | 177 | 169 | 175 | 166 |
| 158 | 156 | 165 | 161 | 162 | 157 | 168 | 163 | 167 | 166 |
|  |  |  |  |  |  |  |  |  |  |


| PANELC |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 171 | 175 | 178 | 168 | 181 | 177 | 185 | 174 | 177 | 177 |
| 169 | 174 | 184 | 173 | 182 | 179 | 178 | 167 | 186 | 175 |
| 176 | 172 | 176 | 174 | 174 | 170 | 184 | 173 | 174 | 174 |
| 179 | 177 | 177 | 176 | 171 | 161 | 172 | 168 | 177 | 176 |
| 186 | 172 | 173 | 184 | 167 | 161 | 166 | 171 | 180 | 163 |
| 181 | 176 | 179 | 176 | 170 | 172 | 165 | 178 | 174 | 182 |
| 169 | 179 | 176 | 183 | 172 | 172 | 170 | 178 | 179 | 178 |
| 179 | 166 | 174 | 184 | 169 | 164 | 177 | 180 | 183 | 172 |
| 183 | 164 | 178 | 166 | 177 | 186 | 174 | 179 | 175 | 179 |
| 183 | 165 | 174 | 173 | 172 | 171 | 176 | 188 | 181 | 169 |


| $\mathbf{P A N E L} \mathbf{D}$ |  |  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 165 | 161 | 168 | 155 | 172 | 160 | 176 | 170 | 162 | 161 |
| 167 | 158 | 155 | 163 | 158 | 159 | 174 | 179 | 161 | 157 |
| 176 | 171 | 160 | 164 | 167 | 173 | 174 | 163 | 162 | 157 |
| 155 | 167 | 161 | 163 | 169 | 168 | 158 | 166 | 160 | 167 |
| 163 | 162 | 165 | 167 | 169 | 161 | 174 | 164 | 154 | 174 |
| 171 | 168 | 162 | 173 | 164 | 172 | 170 | 166 | 165 | 163 |
| 166 | 168 | 158 | 161 | 175 | 164 | 164 | 164 | 167 | 173 |
| 162 | 164 | 161 | 169 | 170 | 157 | 164 | 169 | 161 | 166 |
| 174 | 168 | 174 | 168 | 156 | 160 | 153 | 167 | 167 | 156 |
| 176 | 165 | 161 | 164 | 161 | 163 | 168 | 161 | 173 | 166 |
|  |  |  |  |  |  |  |  |  |  |

[^0]"Homegrown" Exercises around M \& M Chapter 6
-3- Exercise to Illustrate Type I Errors and Statistical Power

## - Birthweight:

Perform a test of each of the following 4 (obviously competing, so not independent) contrasts; use new samples of size $n=4$ and $n=4$ for each of the 4 tests; use a z-test ( $\sigma$ is given) with alpha $=\mathbf{0 . 1 0}$ (two-sided, so $\mathrm{z}_{\mathrm{alph}}=1.645$ ) for each. []

1. $\mu_{\mathrm{A}}$ vs. $\mu_{\mathrm{B}}$
2. $\mu_{\mathrm{C}}$ vs. $\mu_{\mathrm{D}}$
3. $\mu_{\mathrm{A}}$ vs. $\mu_{\mathrm{D}}$
4. $\mu_{\mathrm{B}}$ vs. $\mu_{\mathrm{C}}$

## - Adult heights:

test the following 4 contrasts*, again using $n=4$ vs $n=4$.

1. $\mu_{\mathrm{A}}$ vs. $\mu_{\mathrm{C}}$
2. $\mu_{\mathrm{B}}$ vs. $\mu_{\mathrm{D}}$
3. $\mu_{\mathrm{A}}$ vs. $\mu_{\mathrm{D}}$
4. $\mu_{\mathrm{B}}$ vs. $\mu_{\mathrm{C}}$

* NB: 1 and 2 are not the same as 1 and 2 for birthweight above.

To save you time, the structure of the tests is laid out below.
To help with rapid compilation of results in class, circle below which contrasts yielded "statistically significant" differences and BRING YOUR 8 DECISIONS TO CLASS.

| Birthweights | A vs. B | C vs. D | A vs. D | B vs. C |
| :--- | :--- | :--- | :--- | :--- |
| Adult Heights | A vs. C | B vs. D | A vs. D | B vs. C |

"Arithmetic" of Testing if 2 panels have same mean
$\mathrm{H}_{0}: \mu_{1}=\mu_{2}$ [same sex] $\alpha=\mathbf{0 . 1 0}$ (2-sided) $\quad \mathrm{H}_{\mathrm{alt}}: \mu_{1} \neq \mu_{2}$ [different sexes]
Reject $\mathrm{H}_{0}$ (i.e. infer that $\mu_{1} \neq \mu_{2}$ ) if

$$
\frac{\overline{\mathrm{x}}_{1}-\overline{\mathrm{x}}_{2}}{\sigma \sqrt{\frac{1}{\mathrm{n}_{1}}+\frac{1}{\mathrm{n}_{2}}}}>1.645 \text { or } \frac{\overline{\mathrm{x}}_{1}-\overline{\mathrm{x}}_{2}}{\sigma \sqrt{\frac{1}{\mathrm{n}_{1}}+\frac{1}{\mathrm{n}_{2}}}}<-1.645
$$

(use $z$-test since $\sigma$ is given)
i.e. conclude "different sexes" if

$$
\left|\overline{\mathrm{x}}_{1}-\overline{\mathrm{x}}_{2}\right|>1.645 \sigma \sqrt{\frac{1}{\mathrm{n}_{1}}+\frac{1}{\mathrm{n}_{2}}}
$$

$\sigma$ is given, so we can work out ahead of time (from ***) what difference between $\overline{\mathrm{x}}$ $)_{1}$ and $\bar{x}_{2}$ would lead us to conclude "different sexes"... the average birthweights need to be $>50$ (ie 500 g ) apart, and average heights $>7 \mathrm{~cm}$ apart.
[with t -tests, we don't have $\sigma$, and in fact have to calculate s from sample)

|  | Value of $1.645 \sigma \sqrt{\frac{1}{\mathrm{n}_{1}}+\frac{1}{\mathrm{n}_{2}}}$ |  |
| :---: | :---: | :---: |
|  | BIRTHWEIGHTS $\sigma=43 \mathrm{~g} \mathrm{x} 10$ | ADULT HEIGHTS $\sigma=6 \mathrm{~cm}$ |
| $\mathrm{n}_{1}=\mathrm{n}_{2}=4$ | 50 gx 10 | 7.0 cm |

J ust for inter est, here is what is is for other sample sizes...
$\mathrm{n}_{1}=\mathrm{n}_{2}=8$
35.4 g x 10
4.9 cm
$\mathrm{n}_{1}=\mathrm{n}_{2}=16 \quad 25.0 \mathrm{~g} \mathrm{x} 10 \quad 3.5 \mathrm{~cm}$

On class, I will 'play god' and tell you which contrasts belong in which rows. In practice, you may not be able to unequivocally determine the truth -- or it may take a lot more work. And determining how big a difference is takes even more work.
Results of statistical tests [columns] performed by students in relation to real situations[rows]

| BIRTHWEIGHT | $\begin{gathered} \text { "Can't say" } \\ \text { p >0.10 } \\ \text { ("negative") } \\ \text { ("N.S") } \end{gathered}$ | "different" <br> p<0.10 <br> ("positive") <br> ("Stat. sig.") | $\begin{gathered} \text { No. } \\ \text { of } \\ \text { Tests } \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| same sex |  |  |  |
| different sexes |  |  |  |
| ADULT HEIGHT | $\begin{gathered} \text { "Can't say" } \\ \text { p > 0.10 } \\ \text { ("negative") } \\ \text { ("N.S") } \end{gathered}$ | $\begin{aligned} & \text { "different" } \\ & \text { p<0.10 } \\ & \text { ("positive") } \\ & \text { ("Stat. sig.") } \end{aligned}$ | No. of Tests |


| same sex |  |  |  |
| :--- | :--- | :--- | :--- |
| different sexes |  |  |  |

Results of statistical tests [columns] performed by students in relation to real situations [rows]
BIRTHWEIGHT

|  | $\begin{gathered} \hline \text { "Can't say" } \\ \text { p > } 0.10 \\ \text { ("negative") } \\ \text { ("N.S") } \end{gathered}$ | $\begin{gathered} \text { "different" } \\ \text { p< } 0.10 \\ \text { ("positive") } \\ \text { ("Stat. sig.") } \end{gathered}$ | No. of Tests |
| :---: | :---: | :---: | :---: |
| 3. A vs. D (=) same sex 4. B vs. C (=) |  | AS MPS MS MM MM KR AM BF $8$ | 81 |
| $\text { 1. A vs. B }(\neq)$ <br> different sexes $\text { 2. C vs. D }(\neq)$ |  | $\begin{array}{\|cc\|} \hline \text { BM MS AS KR NW AS } \\ & \\ & \\ & 6 \end{array}$ | 82 |

Results of statistical tests [columns] performed by students in relation to real situations [rows]
ADULT HEIGHT



[^0]:    Key
    Fir [ar clé -- céad, deich is trí fichid, cúig]
    Mná [-- céad, trí fichid, cúig ]

