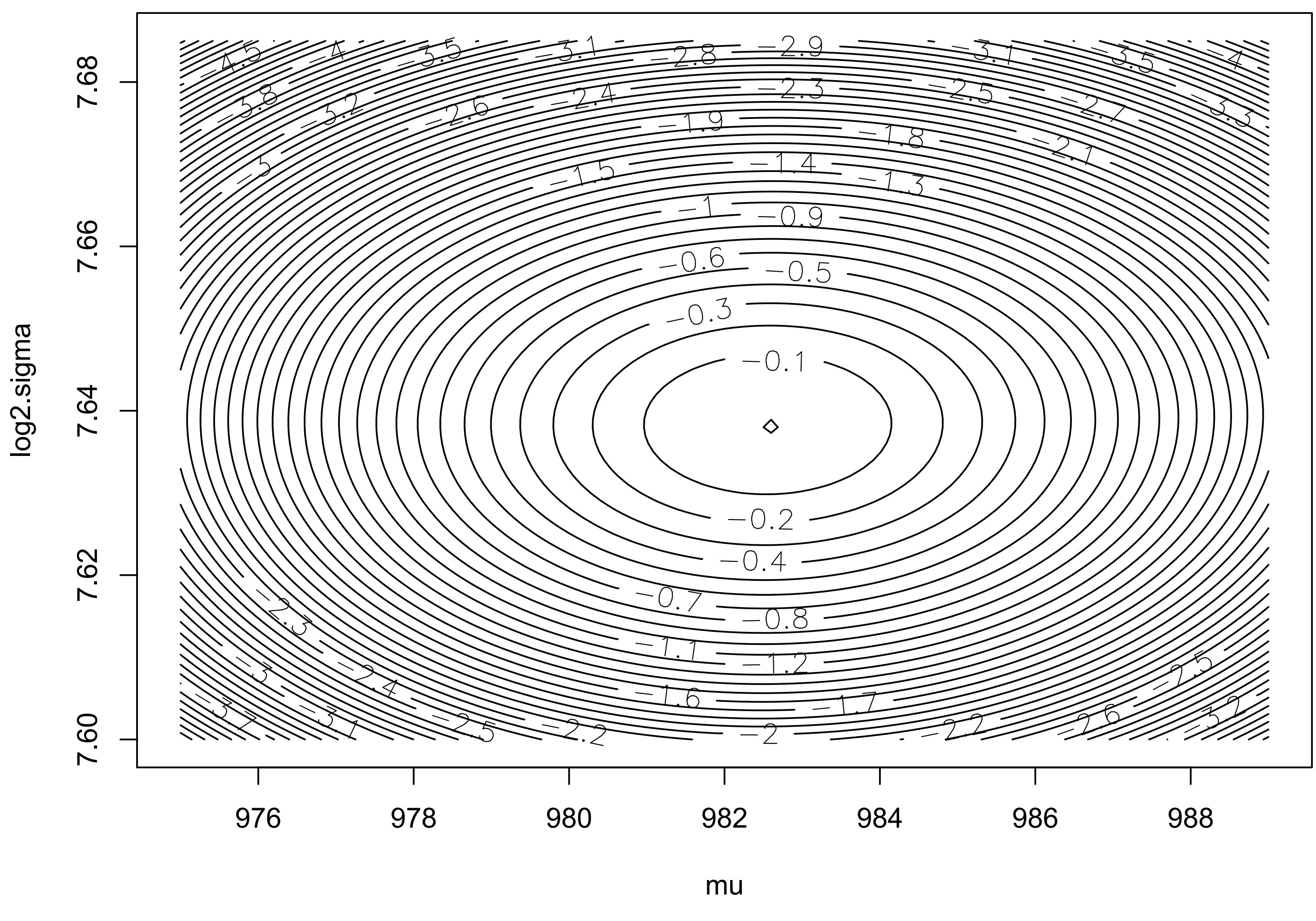
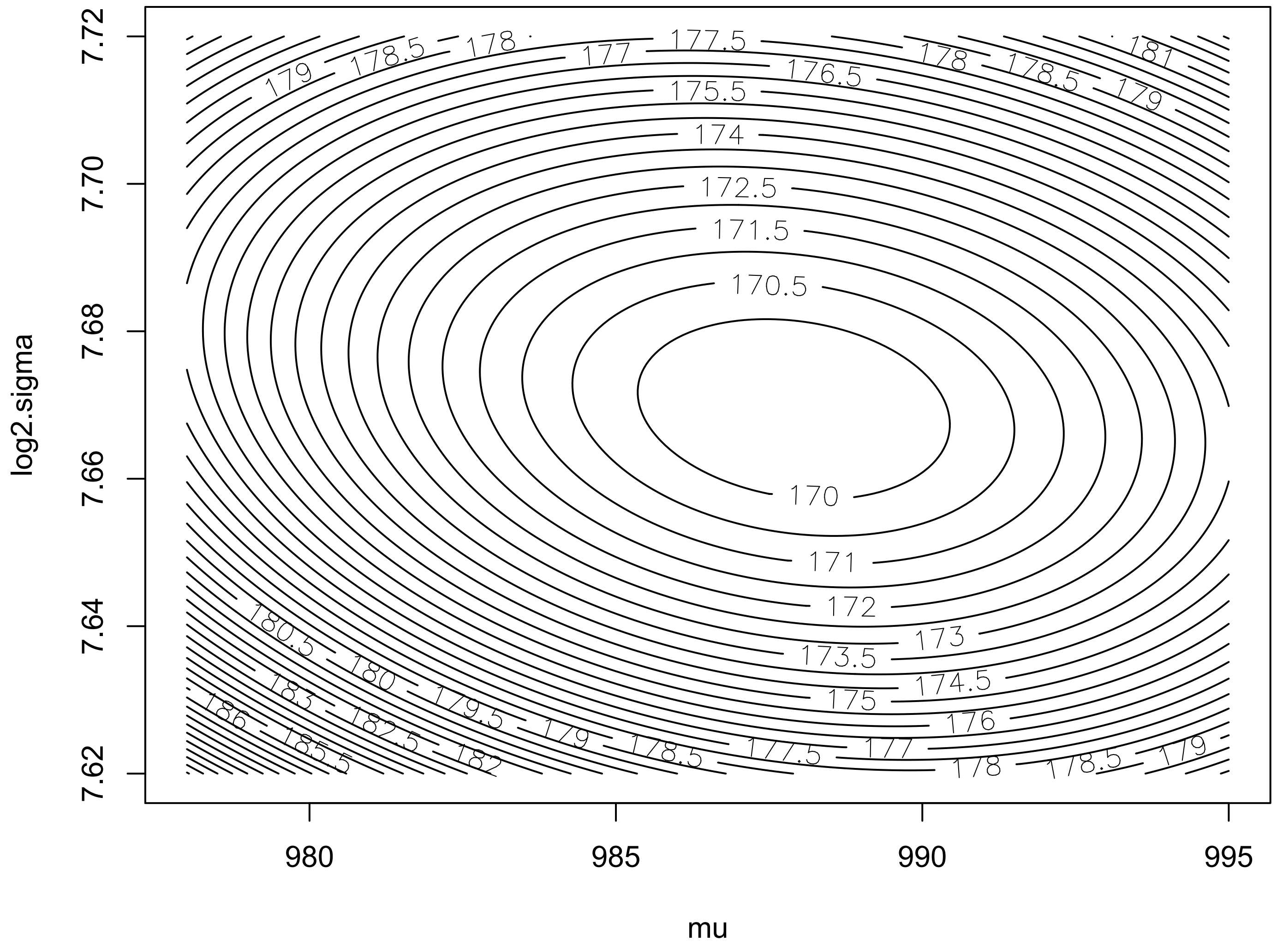
```
> f=scan()
1: 22
2: 43
3: 164
4: 284
5: 598
6: 645
7: 683
8: 396
9: 132
10: 120
11: 120
12:
Read 11 items
> #
>
> f;
 [1]  22  43 164 284 598 645 683 396 132 120 120
> n=sum(f) ; n = sum(f) ; n;   # check
[1] 3207
>
> lower = c(0,seq(500,1400,100)) ; lower
 [1]    0  500  600  700  800  900 1000 1100 1200 1300 1400
> upper = c(seq(500,1400,100),3000) ; upper
 [1]  500  600  700  800  900 1000 1100 1200 1300 1400 3000
>
>
> midpoints = c(475, seq(550,1350,100), 1500)
>
> ave.speed = sum(f*midpoints) / n;  ave.speed
[1] 983.5672
>
> d.f. = n-1
>
> var.speed = sum(f*(midpoints-ave.speed)^2)/d.f. ; var.speed
[1] 40667.57
>
> var.speed  = var.speed  - 100^2/12 ; var.speed ;
[1] 39834.24
>
> sd = sqrt(var.speed) ; sd
[1] 199.5852
>
> se.m = sd/sqrt(n); round(se.m,1)
[1] 3.5
>
> # see notes..
>
> # sd^2 ~ (1/d.f.) * chi.sq.d.f. * sigma^2 ;
>
> # var(sd^2)  = (1/d.f.)^2 * 2*d.f. * sigma^4  ;
> # var(sd)  = var( [sd^2]^(1/2) ) *   [ (d sqrt x / d x )  ]^2;
> # (d sqrt / d x ) = (1/2) x ^(-1/2) ;  (d sqrt / d x )^2 = (1/4) x^(-1)
> # var(sd)  =  (1/d.f.)^2 * 2*d.f. * sigma^4    *  (1/4) * (sigma^2)^(-1)
> #          =  (1/[2d.f.]) *  sigma^2
> # SE(sd)  =  sigma/sqrt(2.d.f.)    ; CV(sd) = 100 / sqrt( 2 d.f.)
>
> se.sd  =  sd / sqrt(2*d.f.) ; se.sd
[1] 2.492479
>
> # limits on sigma
>
> sqrt( var.speed * (1/d.f.) * qchisq(c(0.025,0.975),d.f.)  )
[1] 194.6994 204.4693
>
>
> ###### ML estimates of mu, sigma  # working on log2(sigma) scale..
>
> log.lik = function(mu,log2.sigma)
```

```
+   sum(f*log( pnorm(upper,mu,2^log2.sigma) - pnorm(lower,mu,2^log2.sigma)   ))
>
> log.lik(1000,7)
[1] -7476.13
>
> # this wont work in outer function (since lower upper and f are themselves vectors),
> # so 'force' the issue ...
>
> log.lik.vector = Vectorize( log.lik, c("mu", "log2.sigma") )
>
> mu = seq(900,1100,1) ; log2.sigma= seq(6,9,0.1)
>
> mu = seq(975,989,0.1) ; log2.sigma= seq(7.60,7.685,0.001)
>
> log.lik.values = outer(mu, log2.sigma, log.lik.vector)
>
> max.log.lik.values  = max(log.lik.values)
>
> i.max = apply(log.lik.values==max.log.lik.values,1,sum)
> j.max = apply(log.lik.values==max.log.lik.values,2,sum)
>
> mle = c(mu[i.max],round(2^log2.sigma[j.max],1)) ; mle
[1] 975 194
>
> rel.log.lik.values = log.lik.values  - max.log.lik.values
>
> contour( mu, log2.sigma, rel.log.lik.values, nlevels=40,
+  xlab="mu", ylab="log2.sigma", labcex=1.1)
> chi.sq = function(mu,log2.sigma) {
+       fitted.f = n * (pnorm(upper,mu,2^log2.sigma) - pnorm(lower,mu,2^log2.sigma))
+       return( sum((f-fitted.f)^2/fitted.f) )
+       }
>
> chi.sq(1000,7)
[1] 9840.334
>
> chi.sq.vector = Vectorize( chi.sq, c("mu", "log2.sigma") )
>
> mu = seq(900,1100,1) ; log2.sigma= seq(6,9,0.1)
>
> mu = seq(978,995,0.1) ; log2.sigma= seq(7.62,7.72,0.0005)
>
> chi.sq.values = outer(mu, log2.sigma, chi.sq.vector)
>
> min.chi.sq.values  = min(chi.sq.values)
>
> i.max = apply(chi.sq.values==min.chi.sq.values,1,sum)
> j.max = apply(chi.sq.values==min.chi.sq.values,2,sum)
>
> min.ch.sq.e = c(mu[i.max],round(2^log2.sigma[j.max],1)) ; min.ch.sq.e
[1] 978.0 196.7
>
> contour( mu, log2.sigma, chi.sq.values, nlevels=40,
+  xlab="mu", ylab="log2.sigma", labcex=1.0)
>
```



log2.sigma
mu
7.60
7.62
7.64
7.66
7.68
976
978
980
982
984
986
988
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
−0.8
−0.9
−1
−1.1
−1.2
−1.3
−1.4
−1.5
−1.6
−1.7
−1.8
−1.9
−2
−2.1
−2.2
−2.3
−2.4
−2.5
−2.6
−2.7
−2.8
−2.9
−3
−3.1
−3.2
−3.3
−3.5
−3.7
−3.8
−4
−4.5
−5



log2.sigma
mu
7.62
7.64
7.66
7.68
7.70
7.72
980
985
990
995
170
170.5
171
171.5
172
172.5
173
173.5
174
174.5
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175.5
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176.5
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182.5
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185.5
186