5

The laws of vitality

the laws of vitality are the central points of the science of medicine, and . . . it is only through observations on collective vitality that any precise or numerical knowledge can be obtained representing the laws of individual vitality. The only sure index of the practical success of the science of medicine is in the increase of collective vitality, or in the diminution of collective mortality, without reference to particular diseases.[1]

Like many of his statistician and actuary colleagues – especially Benjamin Gompertz and T. R. Edmonds – William Farr believed not only that certain 'laws of vitality' existed, but that they could be described precisely. What Farr called 'laws' we would now refer to as persistent and general regularities in the structure of mortality. For example, what has come to be known as 'Farr's law' is concerned with the relationship between death rates and population density, while Gompertz and Edmonds are associated with the link between mortality and age, and to some extent sickness and age. Both Gompertz's 'one uniform law of mortality from birth to extreme old age' and Farr's law provide useful pegs on which to hang a general discussion of the structure and variations in mortality in Victorian England and Wales, especially in terms of the ways they were influenced by age and environment. This will also require us to take some account of other rather less law-like regularities in mortality patterns, such as the role of gender. The question of occupational and social variations in mortality will be left to chapter 6 and the 'laws of disease' will reappear in chapter 8.

Age

In the years before the passing of the 1836 Registration Act and the subsequent development of civil registration, actuaries employed by the

[1] Edmonds (1834–35), quoted from p. 5.

life assurance companies were especially concerned to establish the true relationship between the risk of mortality and age. By 1815 they had Richard Price's life table for Northampton and Joshua Milne's for Carlisle, 1778–87.[2] Neither was entirely satisfactory, as Farr subsequently demonstrated, but the latter was at least based on careful surveys not only of age at death but also population at risk distributed by age.[3] It is highly likely that the Northampton table seriously over-estimated the level of mortality and that the Carlisle table may even have under-estimated it, but little could be done to remedy the problem in practical terms until the development of full and accurate national systems of both vital registration and census enumeration. In the 1820s and 1830s several of the most able actuaries turned their attention to the 'laws of mortality' by which they hoped to be able to specify the exact mathematical relationship between the progressive increase in the probability of dying with increasing age and age itself. Once such a law had been specified it could be used to predict mortality given age and to smooth out irregularities in any empirically derived mortality tables.

Benjamin Gompertz is now remembered as the first to provide a formal mathematical description of the law in a paper read before the Royal Society on 16 June 1825.[4] He argued that 'It is possible that death may be the consequence of two generally co-existing causes; the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or an increased inability to withstand destruction', and that mankind was 'continually gaining seeds of indisposition, or in other words, an increasing liability to death'.[5] From this biological base, Gompertz deduced that a form of the following equation could be used to capture the force of mortality:

$$q_x = \mathrm{B} \bullet \mathrm{C}^x \qquad (5.1)$$

[2] Price (1771) and Milne (1815). See O'Donnell (1936), especially pp. 317–79, for an account of the early work on mortality tables. Alborn (1994) considers the actuaries as statisticians.

[3] William Farr, 'The Northampton table of mortality', in Registrar General's *Eighth Annual Report for 1845* (BPP 1847–48/XXV), pp. 290–325. The fourth edition of Price's *Observations* (1783) also contains life tables for Sweden and Stockholm which use data for the number dying at each age and the number alive at those same ages. Milne believed that these were the first life tables to be based on the 'data that are required to determine the law of mortality'. See Milne (1842).

[4] Benjamin Gompertz (1779–1865) was actuary to the Alliance Assurance Company from 1824. For his principal contributions, see Gompertz (1820, 1825, 1862, 1872). The last (Gompertz, 1872) is the transcript of a paper presented at the Fourth International Statistical Congress held in London in 1860. However, Gompertz was not the first to talk about the laws of mortality nor, perhaps, the first to attempt some mathematical formulation. See Dupâquier (1996).

[5] Gompertz (1825), p. 517.

where q_x is the probability of dying at exact age x, and B and C are constants.[6] Of course this would not apply over the full range of x, but from ages 15 or 20 to 50 or 60 Gompertz had certainly succeeded in identifying both a theoretical and practical law of mortality. In 1867 William Matthew Makeham suggested an important modification:

$$q_x = A + B \bullet C^x \quad (5.2)$$

in which the third constant, A, is used to represent the influence of causes of death not dependent on age.

I do not profess to be able to separate the whole category of diseases into the two classes specified – viz., the diseases depending for their intensity solely upon the gradual diminution of the vital power [B], and those which depend on other causes, the nature of which we do not at present understand [A]. I apprehend that medical science is not sufficiently advanced to render such a desideratum possible of attainment at present. I propose only at present to show that there are certain diseases – and those too of a well-defined and strictly homogeneous character – which follow Mr. Gompertz's law far more closely than the aggregate mortality from all diseases taken together.[7]

Makeham was able to offer empirical justification for his modification by using material for the 1850s on age and cause of death reported in the *Supplement* to the Registrar General's *Twenty-fifth Annual Report* (for diseases of the lung, heart, kidneys, brain, stomach and liver) and the *Twenty-sixth Annual Report* (for bronchitis), to show that B did vary with age in a regular fashion and as predicted by Gompertz, but only for certain causes of death. He found it rather more difficult to demonstrate that A was constant with age 'varying only with the peculiar characteristics which distinguish different sets of observations from each other'.[8] Once again, it must be emphasised that the Gompertz–Makeham hypothesis only applies to ages 15 and over.

Apparently independently of Gompertz's elegant but rather abstract

[6] This is the modern equivalent of Gompertz's original notation from his 1825 paper. See Brownlee (1919), Greenwood (1928), Dublin *et al.* (1949), especially chapter 5, 'Biological aspects of the life table', for a more detailed discussion. In June 1927 Raymond Pearl gave a series of lectures at University College London entitled 'Experimental vital statistics'. These became *The Rate of Living, Being an Account of Some Experimental Studies on the Biology of Life Duration* (Pearl, 1928), a work concerned almost exclusively with the fruit fly, but which seems to have stimulated interest among epidemiologists like Major Greenwood who sought to develop parallel models for human populations. But see also Fisher (1930), chapter II, 'The fundamental theorem of natural selection', which uses life table and table of reproduction concepts to discuss evolutionary theory. There has recently been a revival of interest in these issues associated with the problems of ageing; see Olshansky and Carnes (1997) on Gompertz, Makeham, Brownlee, Pearl, etc. on senescence and the 'force of mortality' in old age.

[7] Makeham (1867), quoted from p. 335; see also Makeham (1872). Brownlee (1919), p. 43, refers to this as the 'Gompertz–Makeham hypothesis'.

[8] Makeham (1867), p. 337.

formulations, the young political economist T. R. Edmonds also began to work on the laws of vitality in the early 1830s. Edmonds has not had a law or even a hypothesis named after him, and his work as a political economist has been almost completely ignored, but it is Edmonds who is now credited with having had a particular influence on Farr at just that time when he was acquiring his self-taught statistical knowledge.[9] For example, Eyler argues that 'It was Edmonds's law of mortality which seems to have been Farr's paradigm. In his own statistical studies he hoped ultimately to discover laws analogous to the one Edmonds found in life tables.'[10] Because of their influence on the future work of the Statistical Superintendent as well as their range and substantive content, Edmonds's publications, especially those in *The Lancet* of 1835 and 1836, deserve special attention.[11] As table 5.1 makes clear, merely to list the titles of these works gives an indication of Edmonds's preoccupations and his practical influence.

In *Life Tables* (pp. v–vi), Edmonds outlined his approach to defining the law of mortality.

During the succession of years and moments, measured from the birth of any individual, the continuous change in the force of mortality is subject to a very simple law, being that of geometric proportion. But the same geometric progression is not observed from birth to the end of life. Instead of one there are *three* distinct orders of progression, corresponding to three remarkable periods of animal life. The force of mortality at all ages is expressible, – by the terms of three consecutive geometric series, so connected that the last term of the one series is the first of the succeeding series, – or by the ordinates of three contiguous segments of three log arithmetic curves. The common ratios of the three geometric series (or the constants of the curves) appear to be fixed and immutable, for all human life in all ages of the world. These three constants, now first

9 Thomas Rowe Edmonds (1803–89) is largely unknown today although his early work as a political economist offers an interesting contrast with that of his contemporary, T. R. Malthus. For example, his *Practical Moral and Political Economy; or, the Government, Religion, and Institutions, most conducive to Individual Happiness and to National Power* (Edmonds, 1828) has chapters on 'Population' and 'On the size of towns'. Edmonds's second book, which like Malthus's first was published anonymously, deals with the same problem: *An Enquiry into the Principles of Population, exhibiting a System of Regulations for the Poor; designed immediately to lessen, and finally remove, the evils which have hitherto pressed upon the Labouring Classes of Society* (Edmonds, 1832). Burrow (1966) describes the former as 'quite original, being an attempt to introduce into social thinking the Lamarckian and (Erasmus) Darwinian idea of adaptation' (p. 78).

10 Eyler (1979), p. 76. The extent of Edmonds's influence on Farr can also be judged from the latter's chapter entitled 'Vital statistics; or the statistics of health, sickness, diseases, and death' (Farr, 1837).

11 Edmonds graduated from Trinity College, Cambridge, in 1824 and was appointed actuary to the Legal and General Life Assurance Society in 1832, a post he held for 34 years. In March and May 1835 Edmonds gave his address as Grafton Street, Fitzroy Square, London. Farr lived at 8 Grafton Street between 1833 and 1841 where he is known to have taken in lodgers. See Greenwood (1936), p. 95.

Table 5.1. *The principal publications of T. R. Edmonds on the subject of health and mortality*

Life Tables, Founded upon the discovery of a Numerical Law regulating the existence of every human being: illustrated by a New Theory of the cause producing Health and Longevity (London: James Duncan, 1832)
'On the laws of collective vitality', *The Lancet* 2 (605) (1834–35), pp. 5–8 (28 March 1835)
'On the mortality of the people of England', *The Lancet* 2 (614) (1834–35), pp. 310–16 (30 May 1835), 'Errata', 2 (615) (1834–35), p. 368
'On the law of mortality in each county of England', *The Lancet* 1 (640) (1835–36), pp. 364–71 (7 November 1835) and (641) (1835–36), pp. 408–16 (December 1835)
'On the mortality of infants in England', *The Lancet* 1 (648) (1835–36), pp. 690–94 (18 January 1836)
'On the laws of sickness, according to age, exhibiting a double coincidence between the laws of sickness and the laws of mortality', *The Lancet* 1 (651) (1835–36), pp. 855–58 (13 February 1836)
'On the mortality of Glasgow and the increasing mortality of England', *The Lancet* 2 (667) (1835–36), pp. 353–59 (4 June 1836)
'Statistics of the London Hospital with remarks on the law of sickness', *The Lancet* 2 (679) (1835–36), pp. 778–83 (27 August 1836)
'Corrections of errors in statistics at the British Association', *The Lancet* 2 (680) (1835–36), pp. 837–38 (5 September 1836)
'On the influence of age and selection on the mortality of members of the Equitable Life Assurance Society during a period of sixty-seven years ending in 1829', *The Lancet* 1 (739) (1837–38), pp. 154–62 (21 October 1837)
'On the duration of life in the English peerage', *The Lancet* 1 (754) (1837–38), pp. 705–9 (13 January 1838)
'On the mortality and sickness of soldiers engaged in War', *The Lancet* 2 (765) (1837–38), pp. 143–48 (21 April 1838)
'On the mortality and diseases of Europeans and natives in the East Indies', *The Lancet* 2 (773) (1837–38), pp. 433–40 (2 June 1838)
'The lineage of English peers of ancient titles exhibited by means of diagrams', *The Lancet* 1 (810) (1838–39), pp. 867–73 (February 1839)
'On the mortality and sickness of artisans in London', *The Lancet* 2 (817) (1838–39), pp. 185–93 (April 1839)
'On the mortality of the members of the Amicable and Equitable Assurance Societies', *The Lancet* 2 (994) (1841–42), pp. 839–49 (September 1842)
'On the law of human mortality; and on Mr. Gompertz's new exposition of his law of mortality', *Journal of the Institute of Actuaries* 9 (1861), pp. 327–40

discovered, correspond to the three grand divisions of life, – Infancy, Manhood (or Florescence), and Old Age. For regulating the continuous change in the force of mortality, Nature uses one constant for Infancy, another for Mankind, and a third for Old Age. The constant for *Infancy* confirms life, or indicates a continued diminution of the force of mortality; the constants for *Mankind* and *Old Age* indicate decay of life, or a continued increase in the force of mortality; but the decay of life is much more rapid in the period of Old Age than the period of Manhood.

Edmonds also recognised a fourth period (represented here by p_2) 'between Infancy and Manhood where the force of mortality is stationary and at its minimum' (p. vi). Table 5.2 summarises Edmonds constants – p_1, p_2, p_3 and p_4 – together with the range of ages to which they apply, as well as some of his examples of the level of mortality in various life table populations.[12] In his first *Lancet* paper Edmonds remarked that Dr Richard Price, the author of the Northampton life table, was probably the first to identify the three principal age-components of mortality but that Price did not express the numerical values of the constants.[13] Edmonds also acknowledged in *Life Tables* the prior claim of Gompertz to have first discovered that 'some connexion existed between Tables of Mortality and the algebraic expression (a^{bx})', but he continued to assert that his discoveries had been made independently, that his four constants scheme was more appropriate and that 'The new Theory is *universally* true'.[14] The only exception he allowed was for death in early infancy.

It is only below the age of six weeks that the theory appears to fail, especially for the male sex. The apparent error may be attributable, either to the effects of the act of birth, or to the greater mortality of infants born between the seventh and ninth month from conception. It does not however appear improbable, that the date of conception, and not the date of birth, is the proper commencement of the "infancy period".[15]

12 Edmonds's *Life Tables* also contains some interesting observations on the maximum human birth rate, gender mortality differentials, urban–rural mortality differentials (the combined result of excessive poverty and excessive impurity of air), the poor quality of the recent population returns, and the relation between sickness and death. It concludes with a review of material for the calculation of annuities.

13 'According to Dr. Price [in 1769], human life, from birth upwards, grows *gradually stronger* until the age of 10 years, then *slowly loses* strength until the age of 50, then *more rapidly loses* strength, until, at 70 or 75, it is brought back to all the weakness of the first month.' Edmonds (1861), quoted from p. 327.

14 Edmonds, *Life Tables* (1832), p. xvii. In the early 1860s Edmonds fell into a rather acrimonious dispute with Gompertz's supporters over who had first stated the law of mortality. See Glass (1973), p. 104. In the 1870s other actuaries began to work on the whole mortality curve apparently without reference to Edmonds. For example, Thiele (1872). The publication of this paper led Makeham to have Gompertz (1872) reprinted in the same journal.

15 Edmonds, 'Laws of collective vitality', *The Lancet* (March 1835), p. 6. It is not clear how Edmonds came to this conclusion, but it will be taken up again in chapter 7, pp. 255–61.

Table 5.2. *T. R. Edmonds's rate of mortality constants for specified age ranges and minimum mortality rates for various life table populations*

Constants	Value of p	log p	Age range
p_1	0.6760830	−0.1700	0–8
p_2	1.0	+0.0	8–12
p_3	1.0299117	+0.0128	12–55
p_4	1.0796923	+0.0333	55–end of life
Life table	Minimum mortality	Mortality at birth	Life expectancy at birth in years
Village (Carlisle)	0.005	0.1612228	39.46
Mean	0.00636431	0.1457979	38.69
City	0.00795539	0.1822474	33.01
Northampton	0.009	0.3049598	24.16
Stockholm	0.0127286	0.4313017	15.78

Note:
Edmonds believed that the true age ranges for the Village table (for which he used Carlisle) were 0–9, 9–10, 10–55 and 55–end of life; while for the high mortality corrected Northampton and Stockholm tables they were 0–9, 9–12, 12–62 and 62–end of life. 'City' refers to the largest English towns and cities. Mortality is measured by deaths per year of life.
Source: Edmonds, *Life Tables* (1832), pp. vi and ix.

Edmonds described his law of mortality in the following way:

$$\log y_x = \frac{k^2 \bullet M_0}{\log p}(1 - p^x), \qquad (5.3)$$

where y_x represents the proportion surviving at any age x ($y_0 = 1$); k is the modulus of the common logs. system (0.43429); M_0 is the annual rate of mortality when x is 0; log p is the log of the annual constant rate of increase of mortality and can take on four values depending on x ($p_1, \ldots, p_4$ in table 5.2). Once the lengths of the segments are known and the minimum mortality rate defined, then equation 5.3 may be used to estimate y_x and thereafter l_x given a number born (l_0), and '[f]rom the general formula may easily be deduced an expression for the probability of living one year, at any age; by means of which, Tables of Mortality may be constructed with great rapidity and secure from error'.[16]

Figure 5.1 shows how Edmonds's law of mortality would work using his example of the Mean mortality experience for England and Wales, the constants for which are given in table 5.2.

[16] Edmonds, *Life Tables* (1832), p. xvii. Equation 5.3 is a modified version of the one presented in his *Lancet* paper of April 1839 (p. 191).

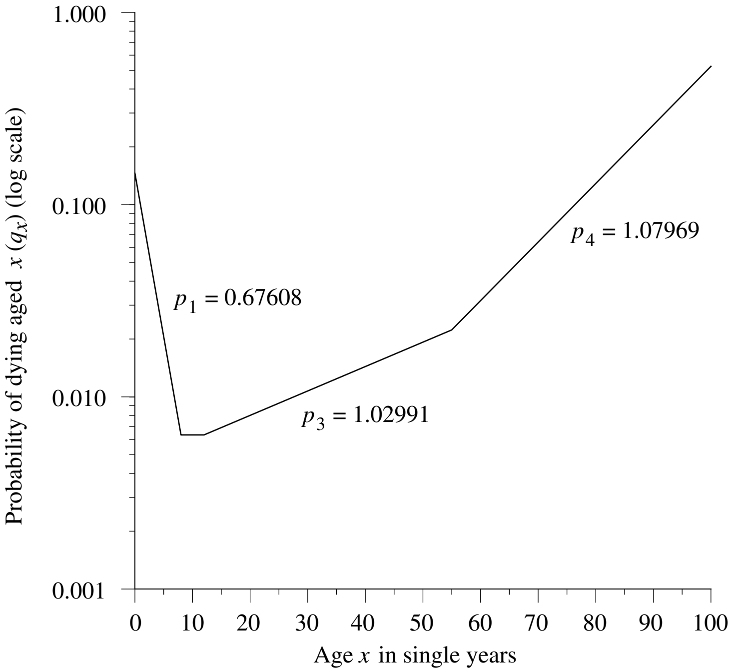

Figure 5.1. Illustration of T. R. Edmonds's law of mortality using his Mean mortality for England and Wales
Note: See table 5.2.

The three- or four-segment curve is an extremely interesting device and, if mortality were to operate in the way Edmonds envisaged, many of the problems faced in representing patterns and constructing life tables would be solved.[17] For actuaries in the 1820s and 1830s it was an especially attractive theory because it allowed the entire mortality curve to be constructed once the minimum rate of mortality had been specified since this value summarised the level of mortality while the constant *p*s determined the actual age-specific rates. Unfortunately, as we

[17] In the 1890s Karl Pearson, apparently independently of the earlier work by the actuaries, identified five distinct age components in the mortality curve: infancy, childhood, youth, middle age and old age. See Pearson (1897). Anson's (1993) work on the shape of mortality curves has placed nineteenth-century England and Wales in the 'troughed' category, as opposed to the 'rectangular'. Had England and Wales been more rectangular than troughed, Edmonds would have found it far more difficult to justify a simple three or four component model. The geographical patterning of age-specific mortality structures has been explored via factor analysis using the registration districts for England and Wales in 1861 by Woods (1982b).

shall see, matters were not quite so simple, but it required the development of a full national system of civil registration and the work of William Farr to show exactly and in detail how mortality did vary with age.

Farr's first attempt to do this, with due acknowledgement to both Gompertz and Edmonds, appeared in an appendix to the Registrar General's *Fifth Annual Report for 1841*. The first English Life Table (ELT 1) was constructed using the 1841 census together with the deaths registered in the same year and the births registered in 1840 and 1841.[18] Prior to the calculation of age-specific death rates Farr made a number of small corrections to the data. The census had been taken on the night of 6–7 June, and in order to estimate the age and sex structure of the population at mid-year (1 July) the totals within each age group were multiplied by $r^{0.07}$ where $r = 1.01334$ (the average annual rate of population increase between 1831 and 1841) and 0.07 was determined for the 24 days between 6–7 June and 1 July by $(24/365 = 0.0658 = 0.07)$. The age structure of the small number of persons enumerated or registered without ages was assumed to be identical to the rest of the population. Finally, deaths were considered to have taken place at equal intervals throughout the year which allowed q_xs to be calculated using the formula $q_x = (M_x)/(1 + M_x)$ where M_x is the age-specific death rate. Once these adjustments had been made, Farr calculated mortality rates for ages 0–4 and then for five-year age groups. He noted:

> Upon a slight inspection it will be seen (1) that the mortality of both sexes decreases until a minimum is attained at the age 10–15; (2), that the mortality increases from 15 to 55 at a slow rate; and (3), that after 55 the mortality is more than doubling every 10 years.[19]

Farr was convinced of the truth of Edmonds's law and consequently he adopted a method of decomposing the mortality rates for five-year age groups into single years by superimposing the law of mortality onto the data, thus ironing out any irregularities. The series of M_xs had shown that the mortality of females aged 25–29 was greater than that of females aged 30–34 and Farr explained this difference in terms of age misreporting caused by the tendency of some to give their ages in decennial increments (i.e. reporting age 30 even though their actual age was between 30 and 39).[20] Farr corrected for this problem by smoothing the

[18] *Fifth Annual Report*, pp. 161–65. 'The following pages contain an account of the methods which were employed in constructing the English Life Table . . . the language employed consists of a very few words the interpretation of which can be easily recovered or acquired' (p. 161). [19] *Fifth Annual Report*, p. 163.

[20] *Fifth Annual Report*, p. 162: 'In practice it was found that neither the ages of the living nor of the dead were stated with sufficient exactness to form the basis of calculations;

two five-year age group series 15–54 and 55–94. Because of the age-heaping, the series 15–54 was treated as two separate geometrical series (15–19, 25–29, 35–39, 45–49 and 20–24, 30–34, 40–44, 50–54) and the ratio between each term was calculated. The geometric mean of these six ratios was then calculated and this formed the common ratio of the corrected series. The first and subsequent terms of the corrected series could then be calculated by assuming that the sums of both the uncorrected and corrected series were equal.[21] Farr now had the necessary raw material to reconstruct his data. Mortality rates for the first five years were based on the actual annual mortality rates in ages 1–4 together with the infant mortality rate, derived from the mean numbers born in 1840 and 1841. For ages 5–9, the M_x for the group was taken to be representative for the ages 7–8 with the other ages being calculated by interpolation, assuming Edmonds's law. It is not clear how Farr treated the age group 10–14, although the shape of the q_x curve suggests that the pattern for the age group 5–9 was simply extended. For the higher age groups it was relatively straightforward to interpolate for the individual years once the overall pattern had been established.[22]

Much of Farr's explanation concerns the practicalities of performing the various necessary calculations required to smooth the data. This involved tedious calculations involving logarithms, with Farr using the difference method to simplify them. Indeed, he realised that many of the calculations could 'I suppose, be calculated by Mr. Babbage's machine'.[23] Farr's method provides the first attempt to construct a national life table for England and Wales. Yet because of the perceived shortcomings of the data, it actually represents a marriage of both the theoretical and practical considerations and in many ways it is a triumph of the application of simple mathematics to the study of populations. However, the pattern of mortality revealed by the first English Life Table is essentially one that owes a great deal to Edmonds's law of mortality.

The *Fifth Annual Report* also contains abridged life tables for three localities: London, non-metropolitan Surrey and Liverpool.[24] An

and if the age had been correctly stated in single years, it would have been necessary to add the numbers together in quinquennial or decennial periods to obtain uniform results.' Figure 2.18 shows that these problems were still present even in 1911.

[21] This can be done using the formula $S_n = a\,(r^n - 1)/(r - 1)$ where S_n = sum to n terms, n = number of terms, a = first term and r = common ratio. In practice, Farr used logarithms to perform these calculations which reduced most of them to a series of additions. I am particularly grateful to Dr Chris Galley for his mathematical advice on this point.

[22] There is a small anomaly in Farr's method since the central series for males was taken from 15 to 55, while that for females from 15 to 54 presumably to provide a better fit with the final series.

[23] *Fifth Annual Report*, p. 167.

[24] See chapter 2, pp. 60–61.