Estimating the Chances of Large Earthquakes by Radiocarbon Dating and Statistical Modeling*

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Residents in earthquake-prone areas are concerned with the possibility that an earthquake might occur and cause them loss of life or property. They seek insurance to reduce the effects of this risk. Government officials are also concerned, for they have the responsibilities of planning continuing services if there is damage to critical facilities and for educating the public. Basic to these insurance premium calculations and government allocation of resources are estimates of the chances of earthquakes and of the associated destruction.

Fortunately large earthquakes are rare. Unfortunately, however, their rarity has the statistical disadvantage of making it difficult to estimate their chances of occurrence confidently. Several procedures have been developed to assess seismic risk. This essay describes a cross-disciplinary approach that has the

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wonderful aspect of being based on data for earthquakes that occurred at a location of interest when no one was there to record the event. In fact, nine of the ten earthquakes employed in the study are prehistoric.

PALLETT CREEK

This story began with Stanford geologist Kerry Sieh heading off into the California desert just after his honeymoon, in the company of his wife and brother. Professor Sieh's destination was a small piece of ground straddling the San Andreas Fault about 55 kilometers northeast of Los Angeles (see Figure 1 for the general location). A stream called Pallett Creek runs nearby. Until 1910 or so this area was a swamp. Over the years, black peats were formed and periodically buried by sand and gravel borne by the creek's floodwaters. Sieh and his companions proceeded to dig trenches. They found disrupted layers of peat, wood fragments, charcoal, and even old animal burrows. Examining the trench walls Sieh noted places where the layers were broken and inferred that these breaks occurred during prehistoric earthquakes. All of his professional train-

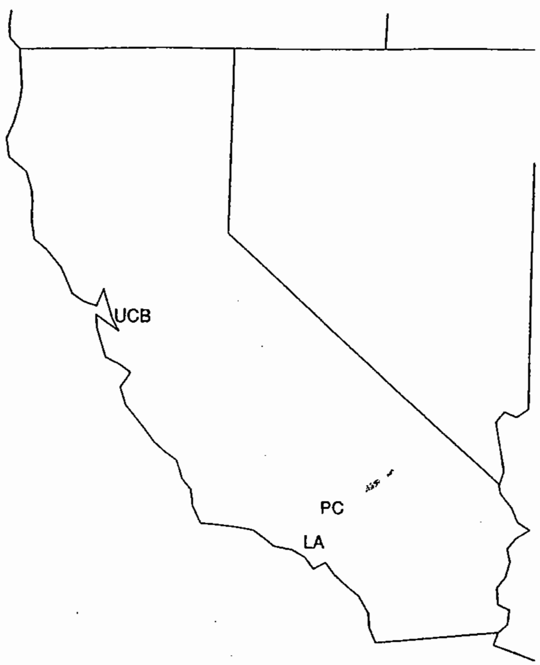

Figure 1 *Map of California showing location of site on San Andreas Fault where specimens were collected for radiocarbon dating. PC indicates the location of Pallett Creek, LA indicates Los Angeles, and UCB the University of California, Berkeley.*

ing and expertise as a geologist helped him to decide which disruptions in the layers might correspond to earthquakes. He selected specimens near each of the breaks to date by radiocarbon techniques.

In the way of background, the most recent large earthquake that affected the Pallett Creek area was in 1857. (A large earthquake is one of Richter magnitude 7.5 or greater.) Also, the study of earthquakes at Pallett Creek is highly informative concerning destructive events that might hit the greater Los Angeles area.

RADIOCARBON ANALYSIS

After returning from Pallett Creek, Sieh sent the specimens to Professor Minze Stuiver at the Quaternary Isotope Laboratory at the University of Washington. Professor Stuiver's job was to provide an estimate of the date at which each specimen was deposited (died). This work is done in two stages. In the first, Stuiver uses a technique called *radiocarbon dating*. This gives him first approximations to the dates at which the specimens died. In the second stage, he uses a calibration technique to improve the approximation. Statistical techniques play important roles in both stages.

At the first stage, Stuiver converts Sieh's specimens to the "purest carbon dioxide in Seattle" and then measures their level of radioactivity. He wants to find out how much of the radioisotope, ^{14}C (radiocarbon), is present in each specimen. He follows a technique set down in 1945 by Professor William Libby at the University of Chicago. Libby knew that living matter, such as a tree, contains a near constant level of ^{14}C during its lifetime. Once the tree dies, however, the ^{14}C decays into another element at a known rate. This is shown in Figure 2. For example, the ^{14}C will be reduced to a half of what it was in 5,568 years and to a fifth of what it was in about 13,000 years. (In Figure 2, the 5,568 is indicated by the solid vertical line. It is referred to as the *half-life*.) So, as Libby saw, it will be possible to find the date of a specimen's death if it is known how much ^{14}C it had originally and how much it has now. The original quantity is impossible to get directly. So at this first stage, Stuiver makes the assumption that the amount of ^{14}C in living material has remained about the same across time and uses a standard material (oxalic acid) to get an estimate of how much radiocarbon there would have been in each of Sieh's specimens at the time of its death. This gives him a proportion. If, for example, the proportion is 0.9, then, using the curve of Figure 2, the time elapsed is approximately 850 years and the corresponding date at which the specimen died is 1987 − 850 = 1137. The year 1137 would be Stuiver's first approximation to the date of the specimen's death. It is referred to as the *radiocarbon date* of the specimen. It should be mentioned that a variety of corrections, for example, for background radiation, are also applied in the course of determining a specimen's radioactivity and radiocarbon date.

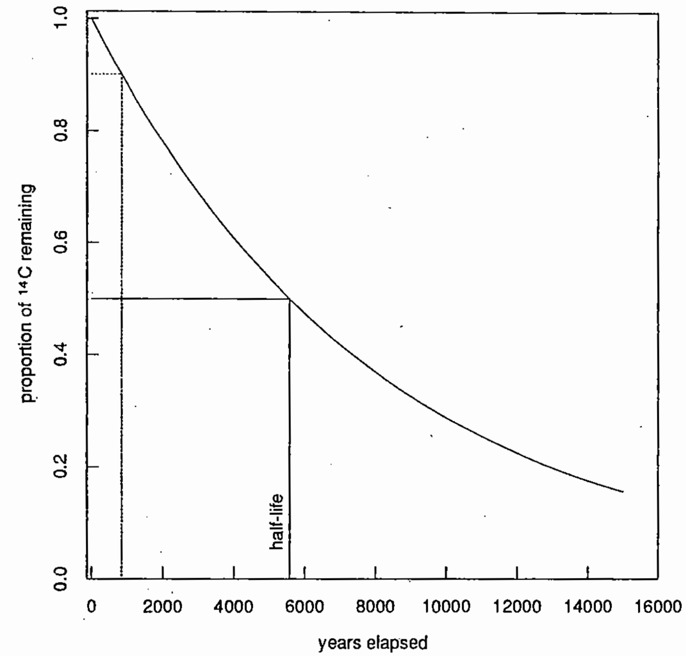

Figure 2 *Plot showing the exponential decay of radioactivity and half-life of radiocarbon. The solid vertical line indicates years passed corresponding to a proportion of 0.5. It provides the half-life of 5,568 years. The dotted vertical line indicates years passed corresponding to a proportion of 0.9.*

CALIBRATION

Professor Stuiver and others have carried out a variety of radiocarbon datings on specimens (tree rings) of known date. They have found that ^{14}C activity in the atmostphere has not remained precisely constant, as Libby initially assumed, but has fluctuated to an extent. Knowing both the radiocarbon and calendar dates of these specimens, the researchers were able to prepare a *calibration curve* relating the two. At the second stage of his work, Stuiver employs such a curve to determine an improved estimate of the calendar date of a given specimen. Figure 3 presents this calibration curve (see Stuiver and Pearson, 1986). For a given radiocarbon date one can read off a calendar date. Suppose one has a specimen with a radiocarbon date of A.D. 1100, then the corresponding calendar date is about A.D. 1200 (see the horizontal and vertical lines in the figure). The calibration operation has been crucial, changing the date by about 100 years.

In his work Stuiver has to deal with measurement errors and to compute estimates of unknown quantities. He also wishes to provide measures of the uncertainties of his estimates. Statistics has a variety of techniques for addressing these problems.

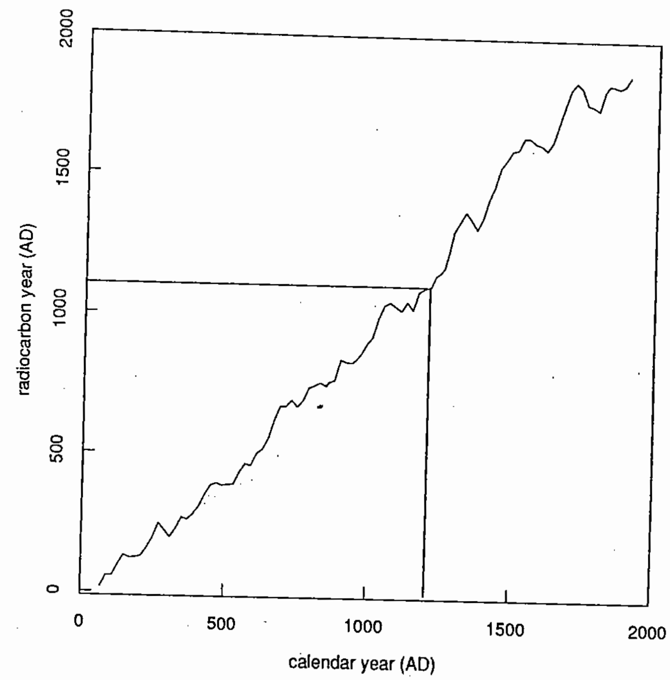

Figure 3 *Calibration curve indicating radiocarbon years and corresponding calendar years. The vertical solid line near 1200 gives the calendar year corresponding to a radiocarbon year of 1100.*

THE STATISTICAL APPROACH

Foremost among the concepts fundamental to the statistical approach to scientific problems is the notion of *distribution*. Supposing that it makes sense to talk of probabilities attached to a circumstance of interest, then the distribution of a numerically valued quantity is the function giving the probability that the quantity takes on a value not greater than a specified number. Figure 4 gives two examples of cumulative distribution functions, a normal and a Weibull. The family of normal frequency distributions have a specific shape—they are symmetrical and have one hump—and they are useful at describing frequency distributions whose observations may represent the sum of many independent contributions such as stature, or scores on achievement tests. The Weibull distributions are a class that have long right-hand tails and have been especially useful for describing the results of life tests, such as time to failure of light bulbs and fatigue tests. More generally, Weibulls can describe the distribution of the time until the next event in a series happens. Figure 4 gives as examples cumulative distributions, and the hump of the normal family is represented by the steep slope near the mean. The long tail of the Weibull is represented by

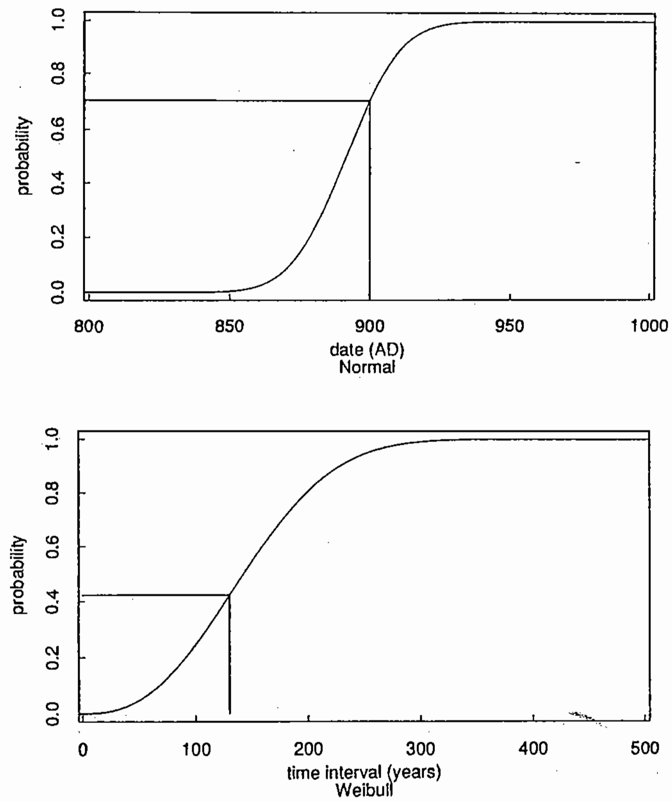

Figure 4 *Examples of distribution functions for two particular distributions (a normal and a Weibull). The curve gives the probability of not exceeding a specified value along the x-axis. In the top graph, the probability corresponding to not exceeding 900 is indicated. In the lower graph, the probability of not exceeding 130 is indicated.*

the slow rise at the right of the figure. From the top graph of Figure 4 one may read that the probability is about .70 of a value, (in this case a date) occurring that is no greater than 900. From the bottom graph, one reads .42 for the probability of a result (in this case a time interval) no greater than 130. Distributions are employed in the construction of *statistical models* (manipulable probabilistic descriptions of situations of concern). With a statistical model, one can address a host of scientific questions in a formal manner.

Distributions generally come in families, individual members of which are labeled by *parameters*. The top graph of Figure 4 illustrates a case of the *normal* family with parameters 891.7 and 15.7, while the lower illustrates a case of the *Weibull* with parameters 2.55 and 164.4. (These particular parameter values are used in calculations with the data later in this essay.) For the normal distribution the first parameter whose value is 891.7 is the mean or average value, and the second parameter is the standard deviation—a measure of spread. (See the essay by Zabell for more detail about the standard deviation.) *In some*

contexts the term standard error is used for the standard deviation of a measure such as a mean or some other observation. Standard error *will be used in the remainder of this essay.* We will not describe the parameters of the Weibull distribution.

In statistical work with data a central concern is choosing an appropriate distribution to employ. *Probability plotting* is one technique for discerning a reasonable family. In probability plotting one graphs on special graph paper an estimate of the distribution function versus a member of a contemplated family. If the family is reasonable, the points plotted will lie near a straight line. Figure 6 (discussed later), provides an example of a Weibull probability plot for the data of concern to Professor Sieh.

Once a family of distributions has been selected, then there is a need to know the parameter values for the best-fitting particular member of the family to the data. This involves *parameter estimation*. Parameter estimation is often conveniently approached via the *likelihood function*. The likelihood function of a hypothesized probability model and given data set is a particular function of the parameters of the model that measures the weight of evidence for the various possible values of the parameters. Figure 5 (discussed below) provides an example with the unknown being the calendar date of one specimen of interest.

After an estimate of a parameter has been found, it is usual to provide some indication of its uncertainty. One particularly convenient means of doing this is via a *95% confidence interval*. These are numerical intervals constructed in such a fashion that over the long run 95% of them will actually contain the true parameter value. The interval of dashes within the curve in Figure 5 is an example of such an interval.

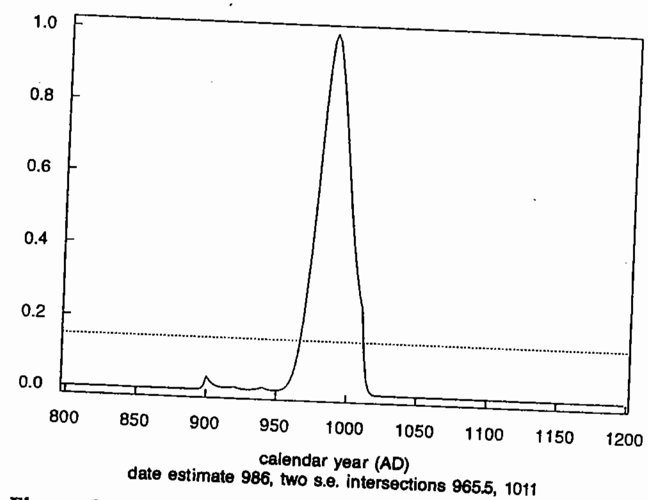

Figure 5 *A plot of the likelihood function corresponding to a specimen of radiocarbon dated 891.7 with a standard error of 15.7. The central peak corresponds to an estimate of the calendar date of the death of the specimen, here* A.D. *986. The dotted line that intersects the curve indicates a 95% confidence interval for the calendar date of the specimen.*

The above statistical concepts* play a role in the analyses described in this essay.

ESTIMATED DATES

Stuiver found one of Sieh's recent specimens to have a radiocarbon date of 891.7, with a standard error of 15.7. Figure 5 provides the likelihood function for the calendar date of deposition of this specimen. In computing this likelihood function the statistician takes into account that both the specimen's radiocarbon date estimate and the calibration curve are subject to measurement errors with approximate normal distributions. The radiocarbon date error depends, in part, on over how long a time period the specimen's level of radioactivity was measured in the laboratory. The calibration curve error depends, in part, on how many known-age items were included in its construction. The date for which the likelihood is largest here is A.D. 986 (see Figure 5). This particular specimen was selected by Sieh to provide a date between the earthquakes that he has labeled I and N in Table 1. By using an interval of twice the standard error on each side of the estimate, we get approximately 95% confidence intervals as shown in Table 1. The 95% confidence interval for the specimen's calendar date runs from A.D. 965 to A.D. 1011. This interval corresponds to the points where the dotted line in the figure intersects the curve in Figure 5. We have no confidence interval for the first entry, 1857, because it is part of recorded history.

In practice there is sometimes an added difficulty. The calibration curve is not steadily increasing as a function of the calendar date. Wiggles appear in it due to things like solar magnetic field disturbances, changes in the Earth's magnetic field, and the measurement error already referred to. The wiggles mean that sometimes one cannot associate a given radiocarbon date with a unique calendar era. To sort out the eras, one needs supplementary information.

INTER-EVENT TIMES

Sieh (1984) lists the following estimated calendar dates for 10 earthquakes at Pallett Creek: 1857, 1720, 1550, 1350, 1080, 1015, 935, 845, 735, and 590. (These are given in Table 1, as well as twice their associated standard errors.) Only one of these dates was available historically, namely, 1857. The other dates were derived by Sieh by interpolation between the estimated dates of the various specimens he selected in the course of his excavations.

At this stage of his study, Sieh turned to a statistician for assistance in inferring the probabilities of future earthquakes. (The radiocarbon daters had turned to statisticians earlier in the development of their estimation procedures.) In the statistical approach to the problem of probability estimation, one seeks a distribution function for the series of times between the events. From the smallest to largest these times are: 65, 80, 90, 110, 137, 145, 170, 200, and 270 years, with 131 years now passed in 1988 since the 1857 event. The statistician sets out to determine a statistical model for these values.

The Weibull family has often been found applicable for the lifetimes of items subject to destruction and for other related phenomena. A Weibull probability plot was prepared for Sieh's data. It is given in Figure 6. The vertical bars correspond to the dating errors of the corresponding interevent times. If the Weibull is adequate for describing the distribution of times between earthquakes, then the points plotted should fall near a straight line. For reference, a straight line has been included in the figure. The Weibull assumption appears reasonable here.

*See Nelson (1982) for an explanation of the ideas of probability plotting, the normal and Weibull distributions, and related statistical concepts.

Table 1 Estimated dates and twice their standard errors for historical earthquakes at Pallett Creek. [The event labels and values are those of Sieh (1984).]

Event	Date, A.D.
Z	1857
X	1720 ± 50
V	1550 ± 70
T	1350 ± 50
R	1080 ± 65
N	1015 ± 100
I	935 ± 85
F	845 ± 75
D	735 ± 60
C	590 ± 55

Source: Sieh (1984).

RISK ESTIMATES

Many people are interested in such questions as: What is the probability of a large earthquake in the Los Angeles area in the next 5 years? In the next 10 years, and so on? These probabilities (risks) may be estimated once one has a distributional form for the times between earthquakes. Figure 7 provides preliminary estimates of risk probabilities, employing the Weibull distribution referred to and using the data of Table 1. From the figure one sees, for example, that the probability of a large earthquake in the next 30 years given that the last earthquake was 131 years ago may be estimated by .32 (that is, there is a 32% chance of one occurring). The dashed lines in the figure provide an indication of the uncertainty in the fitted probability values. They correspond to a 95% confidence interval. By following the horizontal lines of the figure one is led to a lower limit of .18 and an upper limit of .50 for the probability

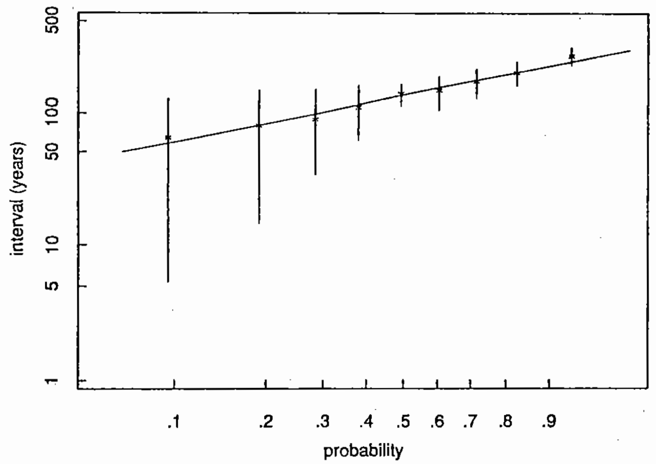

Figure 6 *A probability plot to assess the reasonableness of the Weibull distribution for the intervals between earthquakes at Pallett Creek. The points plotted correspond to the observed intervals. The vertical bars indicate plus and minus twice their standard errors. If the distribution is reasonable the points should fall near a straight line. For reference a fitted line has been added.*

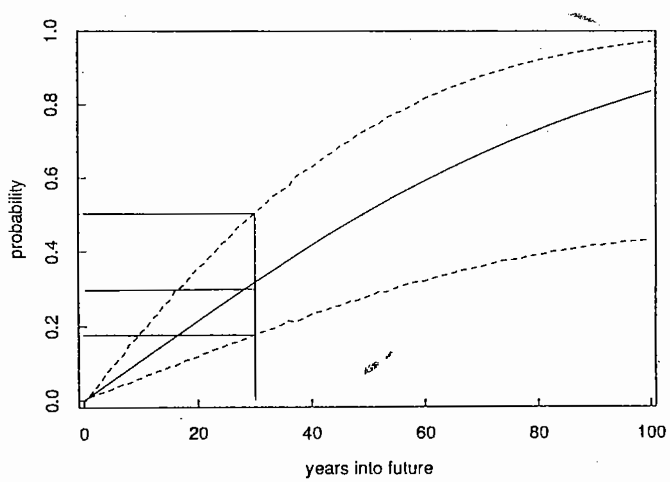

Figure 7 *An estimate of the probability of a future earthquake occurring at Pallet Creek within the indicated number of years. The dashed lines give the upper and lower values of a corresponding 95% confidence interval for the probability. The horizontal lines provide these values for the probability of an earthquake in the next 30 years.*

of an earthquake occurring in the next 30 years. This result may be used by insurers, engineers, and planners in their work.

INSURANCE PREMIUMS

Suppose one wishes to set aside funds to cover the cost of rebuilding a facility that might become damaged in an earthquake in the coming year. The fair premium to cover the rebuilding, were an earthquake to take place, per thousand dollars of cost, is given by a thousand times the probability of an earthquake occurring in the coming year. Using the fitted Weibull, the estimated probability of an earthquake in the coming year is .0108 and the premium works out to be $10.80. (Of course, insurance companies actually "load" their premiums by adding amounts to cover costs, to allow profits, and to protect themselves against extreme catastrophes, so they would charge more than $10.80.)

CONTRIBUTIONS OF STATISTICS TO THIS PROBLEM

The desired end product of a seismic risk study is a probability. So statistics is bound to enter, as statistical distributions are basic to the estimation of probabilities. In the study just described, the tool of radiocarbon dating was crucial. Researchers in that field have long recognized the importance of good statistical techniques. As H. A. Polach (1976) has said, "The application of sound statistical methods has become a radiocarbon dater's 'bread and butter.' "

It is also worth quoting Harold Jeffreys (1967), one of the most important seismologists and statisticians of this century. He has said that "An estimate without a standard error is practically meaningless." This refers to the statement of conclusions. Providing standard errors is one tool for this, confidence intervals are another. These are both central concepts of statistics. Remember that the 95% confidence interval for the probability of a large earthquake at Pallett Creek in the next 30 years runs from .18 to .50.

CONCLUSION

Science proceeds by building on itself. In the work described, specimens of known age (tree rings) are employed to construct a calibration curve that is employed in dating specimens of unknown age. Science uses statistical concepts to address problems of estimating unknowns, to validate assumptions, and to quantify uncertainties in the inferences made.

PROBLEMS

1. Use the curve of Figure 2 to read off the years elapsed for the radiation to drop to a quarter of its initial value.
2. Use the curve of Figure 3 to read off the calendar year corresponding to a radiocarbon year of 500.
3. Use the curve of Figure 3 to find a radiocarbon year that corresponds to several calendar years rather than to a unique year. Comment on this phenomenon.
4. There is a bump around the year A.D. 900 in the curve of Figure 5. What do you think its source is? (Hint: Consider Figure 3.)
5. What is the approximate fair insurance premium to pay to cover $50,000 worth of damage due to an earthquake that might take place in the next 20 years? (Hint: Read a probability estimate from Figure 7.)
6. Evaluate the times between successive events for the earthquakes listed in Table 1. Does there seem to be any structure in the sequence of values?

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