BUGS

Rats: a normal hierarchical model

This example is taken from section 6 of Gelfand *et al* (1990), and concerns 30 young rats whose weights were measured weekly for five weeks. Part of the data is shown below, where Y_{ij} is the weight of the ith rat measured at age x_j.

	Weights Y_{ij} of rat i on day x_j				
	x_j = 8	15	22	29	36
Rat 1	151	199	246	283	320
Rat 2	145	199	249	293	354
....					
Rat 30	153	200	244	286	324

A plot of the 30 growth curves suggests some evidence of downward curvature.

The model is essentially a random effects linear growth curve

$$Y_{ij} \sim \text{Normal}(\alpha_i + \beta_i(x_j - x_{bar}), \tau_c)$$

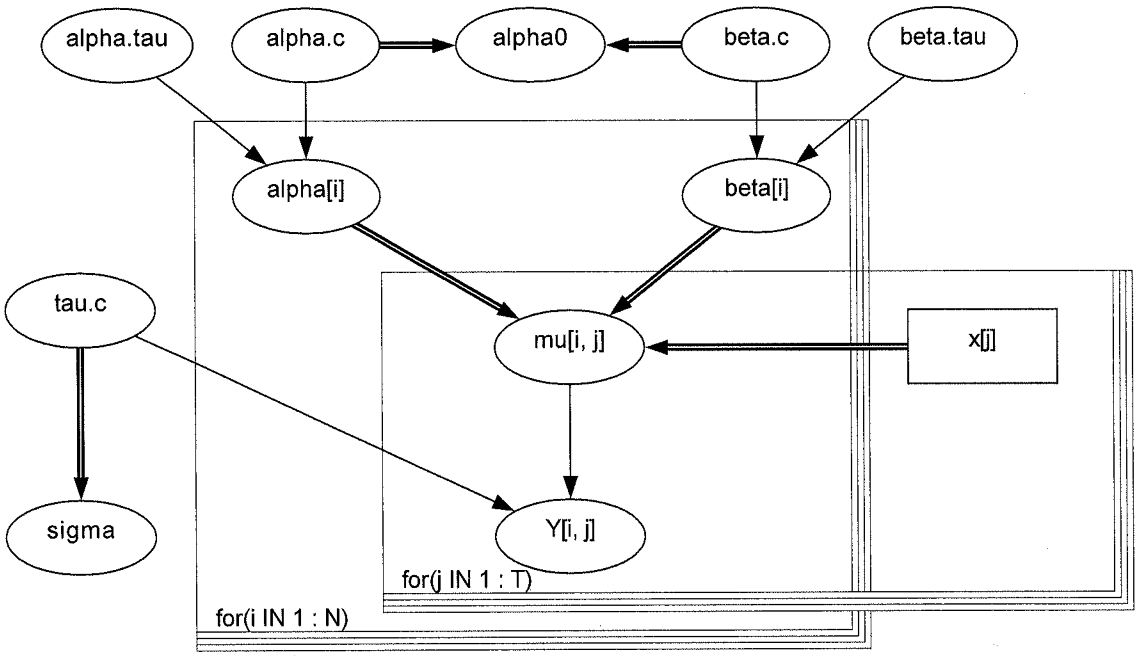
$$\alpha_i \sim \text{Normal}(\alpha_c, \tau_\alpha)$$

$$\beta_i \sim \text{Normal}(\beta_c, \tau_\beta)$$

where x_{bar} = 22, and τ represents the *precision* (1/variance) of a normal distribution. We note the absence of a parameter representing correlation between α_i and β_i unlike in Gelfand *et al* 1990. However, see the `Birats` example in Volume 2 which does explicitly model the covariance between α_i and β_i. For now, we standardise the x_j's around their mean to reduce dependence between α_i and β_i in their likelihood: in fact for the full balanced data, complete independence is achieved. (Note that, in general, prior independence does not force the posterior distributions to be independent).

α_c , τ_α , β_c , τ_β , τ_c are given independent ``noninformative" priors. Interest particularly focuses on the intercept at zero time (birth), denoted $\alpha_0 = \alpha_c - \beta_c x_{bar}$.

Graphical model for rats example:



BUGS *language for rats example:*

```
model
{
    for( i in 1 : N ) {
        for( j in 1 : T ) {
            Y[i , j] ~ dnorm(mu[i , j],tau.c)
            mu[i , j] <- alpha[i] + beta[i] * (x[j] - xbar)
        }
        alpha[i] ~ dnorm(alpha.c,alpha.tau)
        beta[i] ~ dnorm(beta.c,beta.tau)
    }
    tau.c ~ dgamma(0.001,0.001)
    sigma <- 1 / sqrt(tau.c)
    alpha.c ~ dnorm(0.0,1.0E-6)
    alpha.tau ~ dgamma(0.001,0.001)
    beta.c ~ dnorm(0.0,1.0E-6)
    beta.tau ~ dgamma(0.001,0.001)
    alpha0 <- alpha.c - xbar * beta.c
}
```

Note the use of a very flat but conjugate prior for the population effects: a locally uniform prior could also have been used.

Data ⇨list(x = c(8.0, 15.0, 22.0, 29.0, 36.0), xbar = 22, N = 30, T = 5,

```
Y = structure(
   .Data =   c(151, 199, 246, 283, 320,
               145, 199, 249, 293, 354,
               147, 214, 263, 312, 328,
               155, 200, 237, 272, 297,
               135, 188, 230, 280, 323,
               159, 210, 252, 298, 331,
               141, 189, 231, 275, 305,
               159, 201, 248, 297, 338,
               177, 236, 285, 350, 376,
               134, 182, 220, 260, 296,
               160, 208, 261, 313, 352,
               143, 188, 220, 273, 314,
               154, 200, 244, 289, 325,
               171, 221, 270, 326, 358,
               163, 216, 242, 281, 312,
               160, 207, 248, 288, 324,
               142, 187, 234, 280, 316,
               156, 203, 243, 283, 317,
               157, 212, 259, 307, 336,
               152, 203, 246, 286, 321,
               154, 205, 253, 298, 334,
               139, 190, 225, 267, 302,
               146, 191, 229, 272, 302,
               157, 211, 250, 285, 323,
               132, 185, 237, 286, 331,
               160, 207, 257, 303, 345,
               169, 216, 261, 295, 333,
               157, 205, 248, 289, 316,
               137, 180, 219, 258, 291,
               153, 200, 244, 286, 324),
          .Dim = c(30,5)))⇦
```

(Note: the response data (Y) for the rats example can also be found in the file ratsy.odc in rectangular format. The covariate data (X) can be found in S-Plus format in file ratsx.odc. To load data from each of these files, focus the window containing the open data file before clicking on "Data" from the "Model" menu.)

Inits

```
⇨list(alpha = c(250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250,
          250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250, 250),
     beta  = c(6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6,
          6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6),
     alpha.c = 150, beta.c = 10,
     tau.c = 1, alpha.tau = 1, beta.tau = 1)⇦
```

Results

A 1000 update burn in followed by a further 10000 updates gave the parameter estimates:

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
alpha0	106.6	3.625	0.03477	99.32	106.6	113.6	1001	10000
beta.c	6.185	0.1068	0.001354	5.979	6.184	6.398	1001	10000
sigma	6.082	0.4714	0.007308	5.248	6.052	7.093	1001	10000

These results may be compared with Figure 5 of Gelfand *et al* 1990 — we note that the mean gradient of independent fitted straight lines is 6.19.

Gelfand *et al* 1990 also consider the problem of missing data, and delete the last observation of cases 6-10, the last two from 11-20, the last 3 from 21-25 and the last 4 from 26-30. The appropriate data file is obtained by simply replacing data values by NA (see below). The model specification is unchanged, since the distinction between observed and unobserved quantities is made in the data file and not the model specification.

➔ click on one of the arrows to open the data for the missing value analysis 🡐

Gelfand *et al* 1990 focus on the parameter estimates and the predictions for the final 4 observations on rat 26. These predictions are obtained automatically in *BUGS* by monitoring the relevant Y[] nodes. The following estimates were obtained:

node	**mean**	**sd**	**MC error**	**2.5%**	**median**	**97.5%**	**start**	**sample**
Y[26,2]	204.5	8.74	0.1159	187.0	204.4	221.7	1001	10000
Y[26,3]	250.0	10.27	0.1642	229.7	249.9	270.1	1001	10000
Y[26,4]	295.4	12.64	0.2092	270.3	295.3	320.3	1001	10000
Y[26,5]	340.6	15.32	0.284	310.2	340.5	370.5	1001	10000
beta.c	6.575	0.1507	0.003708	6.281	6.573	6.875	1001	10000

We note that our estimate 6.58 of bc is substantially greater than that shown in Figure 6 of Gelfand *et al* 1990. However, plotting the growth curves indicates some curvature with steeper gradients at the beginning: the mean of the estimated gradients of the reduced data is 6.66, compared to 6.19 for the full data. Hence we are inclined to believe our analysis. The observed weights for rat 26 were 207, 257, 303 and 345, compared to our predictions of 204, 250, 295 and 341.