Course EPIB-683 - Intermediate Bayesian Analysis for the Health Sciences

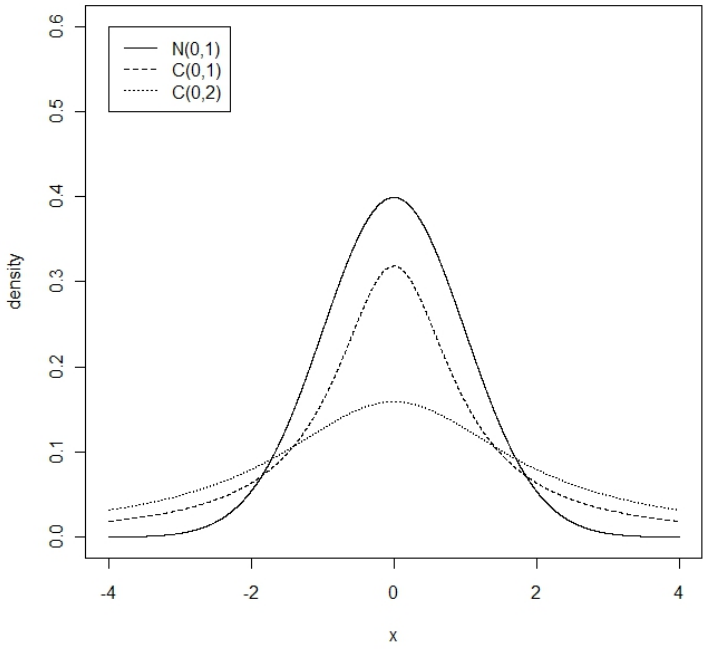
Assignment 1

1. The Cauchy distribution might sometimes be used instead of a normal density when it is thought that the tails of a normal density are too small. The formula for the two parameter Cauchy density is

$$f(x|\mu,\theta) = \frac{1}{\pi\theta\left[1+\left(\frac{(x-\mu)}{\theta}\right)^2\right]}$$

where μ is a location parameter (not the mean, which is in fact not defined for the Cauchy), and θ is a scale parameter.

The graph of the density curve (compared to a standard normal density) is



Consider the data set:

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list(x=c(1.52, 0.30, 1.52, 1.58, -7.62, 0.10, 0.19, 0.98, 2.52, 4.29,
         1.41, 1.78, 2.03, 0.34, -6.22, 1.63, -4.42, 1.32, 0.05, 0.99)
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Use the WinBUGS "ones trick" to estimate μ and θ assuming the data come from a Cauchy distribution, and assuming reasonably wide normal priors for μ and a uniform for θ. You can approximate π as 3.14159. Provide your program and WinBUGS output.

2. Repeat the analyses in question 1 but now use the "zeros trick" rather than the "ones trick" in your WinBUGS programming. Make sure that the inferences from this analysis match those from your program using the ones trick.

3. Suppose there is a standard treatment A and a newly developed treatment B for a certain condition. The success rate of A is believed to be between 65% and 75%, and treatment B, which is more expensive than A, will be considered to be clinically superior to A if its success rate is 10% or greater than the success rate of Drug A.

(a) Construct a prior distribution for the success rate of Drug A in which approximately 95% of the area under the density curve is between 65% and 75%.

(b) Construct an "enthusiastic" prior distribution for the success rate of Drug B, which is centered at 80%, and such that approximately 5% of the prior probability falls below the mean success rate of A (70%).

(c) Similarly, construct a skeptical prior distribution for the success rate of Drug B, centered on the mean success rate of Drug A, and with only 5% of the prior probability falling above the clinical superiority limit of B (80%).

Suppose that a small trial of B is carried out, with 15 successes in 20 subjects for Drug A, and 17 successes in 20 subjects for Drug B.

(d) Estimate the posterior distribution for the difference in success rates using uniform priors for A and B. What is the probability that the success

rate of Drug B is at least 10% larger than the success rate of Drug A? [Hint: Consider use of the WinBUGS "step" command as part of your program.]

(e) Estimate the posterior distribution for the difference in success rates using the enthusiastic prior for B, and the prior for A. What is the probability that the success rate of Drug B is at least 10% larger than the success rate of Drug A?

(f) Estimate the posterior distribution for the difference in success rates using the skeptical prior for B, and the prior for A. What is the probability that the success rate of Drug B is at least 10% larger than the success rate of Drug A?

(g) What is your overall conclusion regarding the choice of Drug A or Drug B, given the posterior distributions calculated in (d), (e) and (f)?

4. Repeat parts (d) through (g) of Question 3, but now suppose that a larger data set is available, with 150 successes in 200 subjects for Drug A, and 170 successes in 200 subjects for Drug B.

5. (a) Again considering the scenario described in Question 3, would you recommend stopping the accumulation of evidence after the smaller trial of 40 subjects?

(b) Now considering the scenario described in Question 4, would you recommend stopping the accumulation of evidence after observing data from the 400 subjects?