# Course EPIB-682 - Intro to Bayesian Analysis for the Health Sciences

## Assignment 1

1. Suppose that two drugs are being compared for the same condition, Drug A and Drug B. Suppose further that two different researchers in two different cities are planning randomized controlled trials of patients with this condition that will investigate which drug is better. Both will use unpaired $t$-tests to make their comparison. Unknown to these researchers, suppose that the null hypothesis is in fact true, i.e., there is in fact no difference in the effectiveness of the two drugs. Both researchers decide in advance to use a type I error of $\alpha = 0.05$. If one researcher uses a sample size of 100, and the other uses a sample size of 1000, does one or the other have a higher probability of rejecting the null hypothesis? Explain your answer carefully.

2. Quality of Life is measured on a five point scale, and is affected by whether one has a certain disease state or not, according to the following table of probabilities conditional on being in each state:

| Quality of life | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Disease Present | .05 | .15 | .4 | .25 | .15 |
| Disease Absent | .01 | .02 | .1 | .12 | .75 |

(a) Suppose the prevalence of the disease in a given population is 10%. If a randomly selected person from the population has quality of life of 3, what is the probability that they have the disease?

(b) Now suppose the disease is rare, occurring in only 1% of the population. If a randomly selected person from the population has quality of life of 1, what is the probability that they have the disease?

3. Suppose that a person claims that they are a psychic, and you decide to test them by having them predict the outcome of a series of coin flips that you will carry out.

(a) Suppose you flip a coin five times, and they in fact correctly predict the outcome (heads or tails) all five times. What is your personal probability that they are in fact a psychic? [Note: there is no "correct" or "incorrect" answer here, I expect all students in the class to give different answers, but there is a correct way to derive your personal probability. Therefore, provide details about where your prior probability came from.]

(b) Same question as in part (a), but re-evaluate now if they get 20 flips in a row correctly.

(c) Now consider the following slightly different situation: A skeptics organization in the US offers a prize of $10 million to anyone who can prove they are a psychic, by correctly guessing the outcome of 20 coin flips in a row. Given the size of the prize, it is not surprising that the contest drew many entrants, all claiming to be psychics, and in fact, there were 1,000,000 such trials carried out on 1,000,000 different people, each claiming to be psychic. Out of these trials, one contestant actually indeed got all 20 flips correct. Do you believe their claim that they are psychic? Explain why or why not.

4. Provide three examples in the analysis and interpretation of data from epidemiological studies where Bayesian methods may be particularly useful. Provide a short paragraph for each situation explaining why Bayesian methods allow for better analyses compared to standard analyses for these situations.

5. Discuss the similarities and differences between the following types of statistical inferences:

(a) Frequentist confidence intervals and Bayesian credible intervals.

(b) Frequentist $p$-values and Bayesian posterior probabilities of a null hypothesis.