Brief Solutions to Final Exam 2000

1. (a) From either z-test ($z = 13.89$) or χ^2 ($X^2 = 192.86$), H_0 is rejected with $p << 0.0001$.
(b) $(-0.8674, -0.7325)$.
(c) From CI, conclude important effect is surely present. The p-value from part (a) adds little if anything to this conclusion.

2. (a) (78.04, 81.96)
(b) True.

3. Posterior 1: $beta(250, 150)$. Posterior 2: $beta(220, 180)$.
(b) from the formulae of the mean of a beta density, $\frac{\alpha}{\alpha+\beta}$, Posterior 1 (mean = 0.625) is closer to the data mean of $180/300 = 0.6$.

4. (a) $a = \overline{Y} - b \times \overline{X} = 65$.
(b) $r = b \times \frac{sd_x}{sd_y} = -0.75$.

5. (a) Slope = 60.71, using standard regression formula for the slope.
(b) For this mouse, between the ages of one to six weeks, there is an average weight gain of about 60 grams per week.

6. From binomial sample size formula, $n = \left(\frac{2\times1.96}{0.15}\right)^2 (0.7)(0.3) = 144$ (rounded up to nearest integer).

7. (a) From the definition of a p-value, expect 1 of the tests to be significant.
(b) Again, from the definition of a p-value, the answer is 0.5.

8. Using Wilcoxon signed rank test, get the sum of ranks is 13 (or 42 in the other group). From table A6, $P > 0.05$ (exact value is 0.084).

9. (a) Have good evidence of a near perfect relationship, but this does not necessarily that the two tests give the same answer (for example, one test could always give half the correct answer).
(b) $t_{100} = \frac{0.92-1}{.002} = 40$, so $p << 0.0001$. Very significant, so surely the slope is not exactly equal to one. However, linear relationship is very strong. By a recalibration based on intercept and slope, could use cheaper test.

10. Sensitivity of test 1 $= \frac{\#\text{positive tests}}{\#\text{true positives}} = \frac{30}{43} = 0.6977$.
Specificity of test 2 $= \frac{\#\text{negative tests}}{\#\text{true negatives}} = \frac{55}{57} = 0.9649$.
(b) From normal approximation to binomial, get (0.5604, 0.8350).

Brief Solutions to Final Exam 1999

1. (a) The zero value for the slope indicates no linear trend of BMD over age for these data. However, from the description, there is a relationship, but rather than linear, it is probably closer to quadratic. Therefore, the zero slope is because of the linearity assumption, and does not in this case indicate no relationship between age and BMD.
   (b) Suggest to analyse the data over smaller age ranges, each piece being linear, or use a more complex model, e.g., quadratic.

2. Paired design, since twins are naturally paired. From Wilcoxon signed rank test, find $p > 0.05$.

3. (a) By subtraction of the α and β parameters, the prior for investigator 1 must have been $\beta(100-75 = 25, 50-25 = 25)$. Similarly for Investigator 2, prior was $\beta(165-75 = 90, 60-25 = 35)$.
   (b) Using formulae for beta distributions, the posterior means are 0.67 and 0.73, which are reasonably close, while the posterior standard deviations are approximately 0.04 and 0.03, which are also close. Therefore, unless a difference from 0.67 to 0.73 is considered as important, these investigators should be close to agreement.

4. Using a χ^2 test, we find $X^2 = 1.9111$, $df = 2$ and $p = 0.3846$. Therefore, there is no evidence to reject H_0.

5. (a) $\widehat{\alpha} = 95.2$ and $\widehat{\beta} = -1.20$
   (b) Slope is amount of MMSE decrease for each one year increase in age, so lose 1.2 points per year, in age range from 62 to 73. Slope is obviously not constant over younger age groups, so that the intercept here is not substantively meaningful. The intercept is used to find the level for the best fitting line, however.

6. (a) From the definition of a p-value, the probability is 0.01.
   (b) From the definition of a p-value, the probability is $0.05 - 0.01 = 0.04$.

7. (a) False, because the definitions of standard deviation and standard error are confused here. The range needs to use SD, not SE.
   (b) True (approximately, uses 2 rather than 1.96).

8. From definition of the mean of a discrete variable, mean $= 0.4 \times (-7) + 0.6 \times 5 = 0.2$. Similarly, variance $= 0.4 \times (-7 - 0.2)^2 + 0.6 \times (5 - 0.2)^2 = 34.56$.
(b)new mean $=$ old mean $\times \frac{1}{3} + 2 = 0.2 \times \frac{1}{3} + 2 = 2.067$. Similarly, new var $=$ old var $\times \frac{1}{3^2} = 34.56 \times \frac{1}{3^2} = 3.84$.

9. (a) Based on formula for a single proportion (normal approximation), and using $z_{1-0.995} = 2.57$, we have the CI is (0.26, 0.34).
(b) Again, use a normal approximation, with mean $= n \times \pi = 300$ and var $= n \times \pi \times (1 - \pi) = 210$. Probability is approximately 0.5156.

10. (a) From χ^2 tables, the probability is 0.2.
(b) From Normal tables, the probability is 0.1056.

Brief Solutions to Final Exam 1998

1. (a) On average, these subjects gained 3 Kg per year.
(b) No, probably not the same curve for young persons, who may gain weight much more rapidly as they grow.
(c) (i) weight gain in this age range may actually be less than in younger subjects, or (ii) random variation, as this estimate is based on only 20 subjects.

2. Combining data provides a larger sample size, implying more accurate estimates of all parameters. However, the range of BMD in men and women are very different, and decline in BMD seems larger in women. Therefore, overall probably not a good idea to combine, better to fit a different line for men and women. [Admittedly, this is hard to see in current format...graphics were much clearer on original exam copies!]

3. (a) Posteriors are beta(780,320) and beta(705, 305).
(b) Very similar, as posterior means and variances are very similar.

4. Clearly a paired design, so Wilcoxon signed rank test will be used. Using this test, find $p > 0.05$, so no evidence to reject null hypothesis.

5. (a) Intercept = –98.33, slope = 6.5.
(b) On average, total cholesterol goes up by 6.5 for each unit increase in HDL cholesterol, over the ranges given.

6. (a) Using standard sample size formula for binomial, get $n = 1111$.
(b) As no longer a purely random sample, more complex variance formulae will be needed. Also, absences within classes and schools may be correlated with each other (i.e., not independent), so further adjustments may be needed.

7. (a) False, confuses SD and SE.
(b) True.

8. (a)Estimated intercept $= a = \overline{Y} - b \times \overline{X} = -5 + 2 = 3$.
(b) $r = b \times \frac{SD_x}{SD_y} = (-1)(1/0.75) = -1.33$, which is impossible, so there is a typo in the original question!! It should have been that $SD_x = .1$ rather than 1, so that $r = (-1)(0.1/0.75) = -0.133$.

9. The answer is a 50% probability, since we will be in the middle of the distribution in terms of the cut-point for significance.

10. From the definition of a p-value, we need to calculate the probability that we get 1 or more successes in 10 trials, where the probability of success is 0.05. This is the same as 1 minus the probability of exactly zero successes. Directly from the binomial distribution, we see that $Pr\{0\ successes|Bin(0.05, 10)\} = 0.5987$, so our p-value $= 1 - 0.5987 = 0.4013$.

Brief Solutions to Midterm Exam 2000

1. We are testing $H_0: \mu_1 = \mu_2$ versus $H_A: \mu_1 \neq \mu_2$. We use an unpaired t-test with degrees of freedom 6998 or 3499, depending on whether we assume equal variances or not (both can be defended, since observed variances are similar, but there is no strong theoretical reason to expect they will be identical). In either case, the normal approximation to the t distribution can be used, since degrees of freedom are so large. Calculating the $t = 2.09$ and looking up on normal tables, we find $p = 0.0366$. Thus we find statistical significance ($p = 0.0366$), but the difference is probably not of much clinical interest.

2. (a) Using the CI from an unpaired t with $df = 15$, we find the CI $= (-10.4, 14.4)$.
(b) Result is inconclusive, since the CI includes both zero and values that would be considered as clinically interesting.

3. The prior is $N(\theta = 75, \tau^2 = 25)$, and the data variance is $\sigma^2 = 100$. We observe $\overline{x} = 80$ and $n = 3$. Thus the posterior distribution is $N(77.14, 14.285)$, using the formula for updating normal means via Bayes Theorem, as given in the course notes.

4. (a) Using results for linear combinations of Normal random variables, we have $N(\mu = 0, \sigma^2 = 34)$. (b) Similarly (or applying rule for means), we find $N(\mu = 0, \sigma^2 = \frac{1}{3})$.

5. (a) Using the probability rule for complements,

$$Pr\{X \geq 2\} = 1 - Pr\{X = 0\} + Pr\{X = 1\} \ .$$

Plugging into the Poisson formula and subtracting gives 0.5940.

(b) Use the Normal approximation to the binomial distribution. The mean and variance of the closest fitting Normal distribution is $\mu = 12$) and $\sigma^2 = 84$. Following the usual steps for normal approximations (including the continuity correction) gives 0.0398. [Exact answer by binomial formula is 0.0435, which is reasonably close to approximate solution.]

6. For diagnostic testing situations, Bayes Theorem can be expressed as

$$PPV = \frac{sensitivity \times prevalence}{sensitivity \times prevalence + (1 - specificity)(1 - prevalence)}$$

Plugging in what is known in this case, we have:

$$0.6 = \frac{0.9 \times prevalence}{0.9 \times prevalence + (1 - 0.8)(1 - prevalence)}$$

Algebraically solving for the prevalence gives prevalence=0.25.

Brief Solutions to Midterm Exam 1999

1. (a) The drug seems effective, but it is unclear as to whether the degree of effectiveness in within the range of clinical equivalence or not. Values both inside and outside of the range of clinical equivalence are included in the CI. Further experimentation is required to reduce the width of the CI so that more definitive and clinically relevant decisions can be made. (b) Total width of the 95% CI was 14, so that the SE is approximately $14/4 = 3.5$. Thus the SD must have been close to $\sqrt{25} \times 3.5 = 17.5$.

2. We use a one sample t-test. We are testing $H_0: \ \mu = 10$ versus $H_A: \ \mu \neq 10$. We use the t-test with degrees of freedom 19. Calculating the $t_{19} = \frac{8-10}{3/\sqrt{20}} = -2.98$ and looking up on t_{19} tables, we find $p = 0.0077$ (two-sided). Thus we can reject H_0, the clotting times seem different.

3. Easier to use the rule of complements, i.e., find the probability of no markers, and subtract this answer from one.

$$Pr\{\text{no marker present}\} = (1 - 0.4) \times (1 - 0.6) = 0.24$$

   So there is a $1 - 0.24 = 0.76$ chance of at least one marker present.

4. For diagnostic testing situations, Bayes Theorem can be expressed as

$$PPV = \frac{sensitivity \times prevalence}{sensitivity \times prevalence + (1 - specificity)(1 - prevalence)}$$

   Plugging in what is known in this case, we have:

$$0.5 = \frac{sensitivity \times 0.2}{sensitivity \times 0.2 + (1 - 0.8)(1 - 0.2)}$$

   Algebraically solving for the sensitivity gives 0.8, or 80% sensitivity.

5. By the probability rules for normal distributions, we have $N(\mu = 12, \sigma^2 = 8)$.

6. (a) From normal tables (after standardizing) 0.1587. (b) 0.15, directly from t tables with 30 df.

7. From the normal approximation to the binomial (with continuity correction), we find $Pr\{X \geq 190\} \approx 0.0126$. Note that the mean = 180 and variance = 18 in this approximation.

8. Prior: Summarizes the (subjective assessment of the) information about the parameter(s) of interest before the current data become available. Here, it is believed a priori that the average time to tiring is between 15 to 25 minutes.

   Likelihood: Summarizes the information contained in the current data set. Here we learn that $\overline{x} = 26$, and the data say that the mean time to tiring is likely to be around 25 to 28 minutes.

   Posterior: Combines the information from the prior and likelihood, to show what we should now believe, having observed the data, and starting from our prior distribution. Here the average time to tiring is between about 24 to 27 minutes.

9. Using the formulas for the mean and variance for discrete variables from section 3 of the course, we find that the average is 2.1, and the variance is 0.69.

10. Using the formula for sample size for confidence intervals, we find

$$n = \frac{z^2\sigma^2}{d^2} = \frac{1.96^2 * 1.8^2}{(0.05)^2} = 4979$$

(after rounding up).

Brief Solutions to Midterm Exam 1998

1. (a) From normal tables, probability = .1587 (b) From normal tables, probability= .0436.

2. Using the rules of probability, we calculate $0.85^5 \times 0.15 = 0.06656$.

3. Using the definitions of means and variances for discrete distributions, we have (a) 0.61 cats, and (b) 0.4779 cats.

4. (a) Stress Test Delta will have more false positive results, since the specificity is smaller. (b) Using Bayes theorem (or a two-by-two table), we find the positive predictive value of Stress Test Alpha is .3871.

5. Using a one sample t distribution with 100 degrees of freedom, we find the 95% CI is (193.72, 212.28).

6. We use a two sample t-test with unequal variances and therefore degrees of freedom of 100. We are testing $H_0: \ \mu_1 = \mu_2$ versus $H_A: \ \mu_1 \neq \mu_2$. Calculating the $t_{100} = \frac{1-0.96}{\sqrt{\frac{0.1^2}{101}+\frac{0.15^2}{201}}} = 2.75$ and looking up on t_{100} tables, we find p=0.007 (two-sided). Thus we can reject H_0, there is evidence that the BMD's differ in the two groups.

7. Using the two sample normal formula for sample size via confidence interval widths, we find $n = 768$, after rounding up.

8. (a) Using the binomial distribution, probability = 0.5905 (b) Using the normal approximation to the binomial distribution (with continuity correction), probability $\approx$ 0.0199.

9. Prior: Summarizes the (subjective assessment of the) information about the parameter(s) of interest before the current data become available. Here, it is believed a priori that the average expenditure is between 180 to 280.

   Likelihood: Summarizes the information contained in the current data set. Here we learn that $\overline{x} \approx 255$, and the data say that the mean expenditure is likely to be around 230 to 270.

   Posterior: Combines the information from the prior and likelihood, to show what we should now believe, having observed the data, and starting from our prior distribution. Here the average expenditure is between about 230 to 260.

10. We are virtually certain that Drug B is better, but there is still a chance that the true difference may not be highly clinically relevant. I would use Drug B until further data were available.